METE 3100U
Actuators and Power Electronics

Lecture 11
Rotating Machines

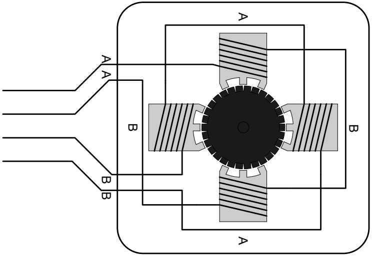
Outline of Lecture 11

By the end of today's lecture, you should be able to

- Understand the working principle of rotating machines
- Model a rotating machine
- Calculate the torque developed in a simple motor

Applications

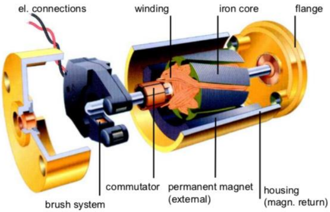
How can the concept of linear electromagnets be extended to create a rotating machine?

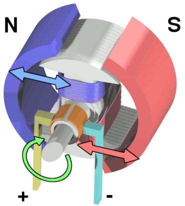


How can the torque be calculated?

Applications

What is the difference between an induction motor and a DC motor?

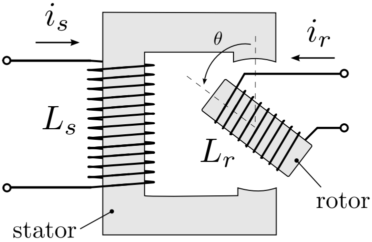




Rotating machines

Stator: Fixed part - subscript *s*

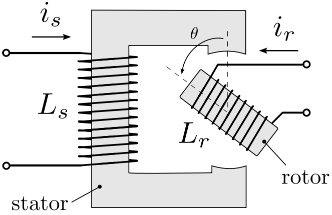
Rotor: Rotating part - subscript *r*



The stored field energy (no motion condition):

$$dW_f = e_s i_s dt + e_r i_r dt$$
$$dW_f = i_s d\lambda_s + i_r d\lambda_r$$

Rotating machines



For a linear magnetic system $\lambda = iL$, thus

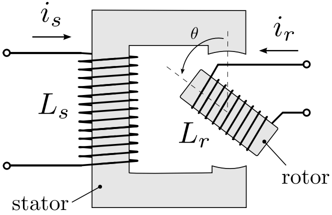
$$\lambda_s = L_s i_s + L_m i_r$$
$$\lambda_r = L_r i_r + L_m i_s$$

L_m is the mutual inductance

A stator current i_s induces an emf e_{rs} in the rotor

$$e_{rs} = -N_s \frac{d\Phi_s}{dt} = -\frac{d}{dt}\int\int \vec{B_s} \cdot d\vec{A} = L_m \frac{di_r}{dt} \quad (1)$$

Rotating machines



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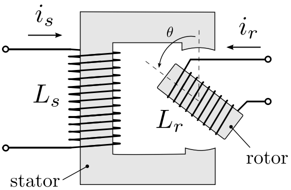
Rotating machines



The store energy as a function of the inductances is

$$dW_f = i_s d(L_s i_s + L_m i_r) + i_r d(L_r i_r + L_m i_s)$$

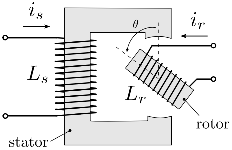
$$dW_f = L_s i_s d(i_s) + L_r i_r d(i_r) + L_m \underbrace{(i_s di_r + i_r di_s)}_{d(i_s i_r)}$$

$$dW_f = L_s i_s d(i_s) + L_r i_r d(i_r) + L_m d(i_s i_r)$$

The field energy is

$$W_f = L_s \int_0^{i_s} i_s di_s + L_r \int_0^{i_r} i_r di_r + L_m \int_0^{i_s} di_s \int_0^{i_r} di_r$$

Rotating machines



The field energy is

$$W_f = L_s \int_0^{i_s} i_s di_s + L_r \int_0^{i_r} i_r di_r + L_m \int_0^{i_s} di_s \int_0^{i_r} di_r$$

$$W_f = \frac{1}{2} L_s i_s^2 + \frac{1}{2} L_r i_r^2 + L_m i_s i_r$$

Recall that

$$L(\theta) = \frac{N^2}{\mathcal{R}(\theta)} \tag{3}$$

Thus, the torque developed in the actuator is

$$T(\theta, i) = \left. \frac{\partial W_f'(i, \theta)}{\partial \theta} \right|_{i=cte} \tag{4}$$

and it can be calculated as

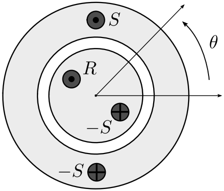
$$T(\theta, i) = \frac{1}{2} i_s^2 \frac{dL_s}{d\theta} + \frac{1}{2} i_r^2 \frac{dL_r}{d\theta} + i_s i_r \frac{dL_m}{d\theta} \tag{5}$$

Cylindrical machines

→ Windings are distributed over several slots

→ It can be assumed that the self-inductances are constant

→ The mutual inductance varies with rotor position



The torque reduces to

$$T(\theta, i) = i_s i_r \frac{dL_m}{d\theta} \quad (6)$$

Now, let

$$L_m = M \cos\theta \quad (7)$$

where M is the peak value of the mutual inductance L_m

Cylindrical machines

Let the stator and rotor currents be

$$i_s = I_s \cos(\omega_s t)$$
$$i_r = I_r \cos(\omega_r t + \alpha)$$

where ω_s and ω_r are current frequencies. The position of the rotor as

$$\theta = \omega_m t + \delta \tag{8}$$

where ω_m is the rotor speed and $\delta = \theta_{t=0}$

The torque becomes

$$T = -I_s I_r M \cos(\omega_s t)\cos(\omega_r t + \alpha)\sin(\omega_m t + \delta)$$

Case 1 $\omega_r = 0, \alpha = 0, \omega_m = \omega_s$: synchronous speed

$$T = -\frac{I_s I_r M}{2}[sin(2\omega_s t + \delta) + \sin(\delta)] \tag{9}$$

This is the principle of operation of synchronous machines.

Cylindrical machines

Case 2 $\omega_m = \omega_s - \omega_r$: asynchronous speed ($\omega_m \neq \omega_s \neq \omega_r$)

$$
\begin{aligned}
T = -\frac{I_s I_r M}{4} & \sin(2\omega_s t + \alpha + \delta) \\
& + \sin(-2\omega_r t - \alpha + \delta) \\
& + \sin(2\omega_s t - 2\omega_t rt - \alpha + \delta) \\
& + \sin(\alpha + \delta)
\end{aligned}
$$

This is the principle of operation of asynchronous machines.

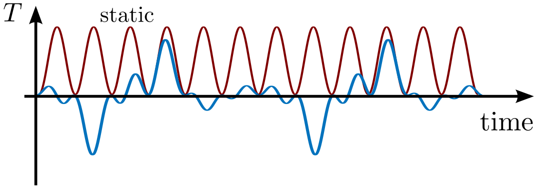
The average torques are

→ Synchronous: $T_s = -\frac{I_s I_r M}{2} \sin\delta$

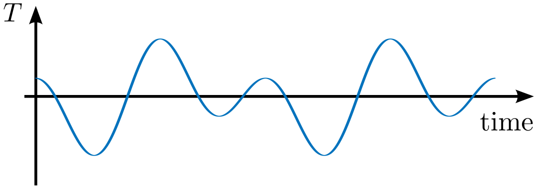
→ Asynchronous: $T_a = -\frac{I_s I_r M}{4} \sin(\alpha + \delta)$

Torque characteristics

Sinusoidal current supply



DC current supply

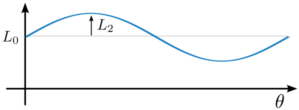


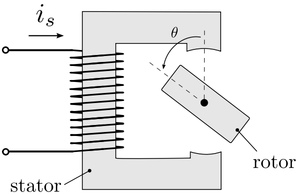
Exercise 47

In the electromagnetic system shown, the rotor has no windings and the stator inductance as a function of the rotor position θ is

$$L_s = L_0 + L_2 \cos(2\theta)$$

as shown in the left.



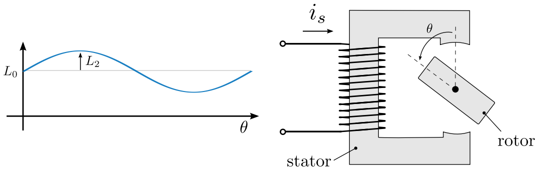


If the stator current is

$$i_s = I_s \sin(\omega t),$$

calculate the torque developed in the motor.

Exercise 47 - continued

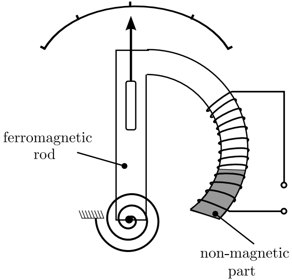


Exercise 48

The current meter shown has a ferromagnetic rod that is pulled into the solenoid when a current flows through the coil. The inductance of the coil is

$$L(\theta) = 4.5 + 180\theta \ \mu H$$

where θ is the angle of deflection in radians. The spring constant is 0.65×10^{-3} Nm/rad.

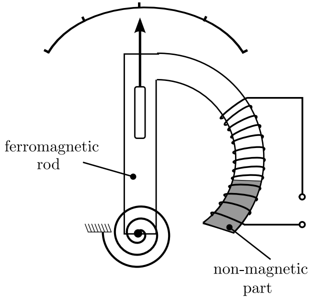


(a) Show that the sensor measures the rms value of the current

(b) For a current of 10 A, determine the angular deflection

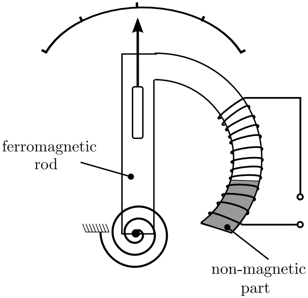
Exercise 48 - continued

(a) Show that the sensor measures the rms value of the current



Exercise 48 - continued

(b) For a current of 10 A, determine the angular deflection

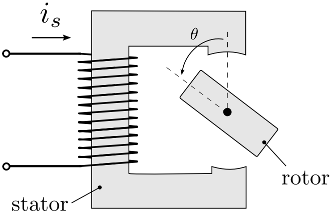


Exercise 49

The inductance of the reluctance machine coil is giving by

$$L_s = 0.1 - 0.3\cos(2\theta) - 0.2\cos(4\theta)$$

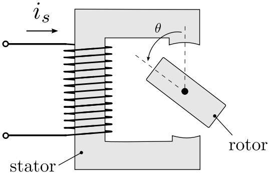
measured in Henry. A current of 10 A at 60 Hz is passed through the coil.



(a) Determine the torque equation if the rotor position is $\theta = \omega t + \delta$

(b) Plot the torque as a function of the rotor speed using Matlab

Exercise 49 - continued

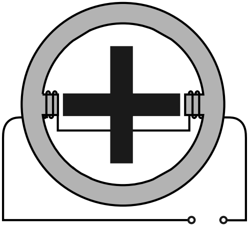


Exercise 50

The reluctance of the 4-pole reluctance motor can be assumed to be a sinusoidally varying function of the angular position θ as follows

$$\mathcal{R}(\theta) = 2 \times 10^5 - 10^5 \cos(4\theta).$$

The coil has 200 turns and negligible resistance and is connected to a 120 V (rms), 60 Hz, single-phase supply.

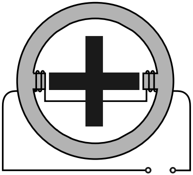


(a) Obtain the expression for the magnetic flux as a function of time.

(b) Show that the torque developed is $T = \frac{1}{2}\Phi^2 d\mathcal{R}/d\theta$

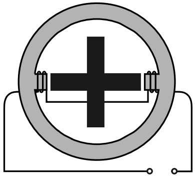
Exercise 50 - continued

(a) Obtain the expression for $\Phi(t)$.



Exercise 50 - continued

(b) Show that the torque developed is $T = \frac{1}{2}\Phi^2 d\mathcal{R}/d\theta$

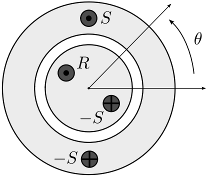


Exercise 51

The self inductance in the rotating machine can be modelled as

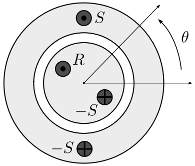
$$L_m = 0.08\cos(\theta)\ \text{H}$$

The rotor rotates at 3600 rpm and the stator carries a current of 5 A (rms) at 60 Hz.

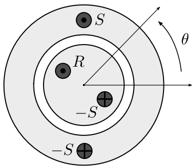


Determine the instantaneous voltage and rms voltage induced in the rotor coil.

Exercise 51 - continued



Exercise 51 - continued



Next class...

- Stepper motors

Additional supporting materials for Lecture 11:

Mutual inductance: https://goo.gl/U8haXD

Workings of synchronous motors: https://youtu.be/Vk2jDXxZIhs

Workings of induction motors: https://youtu.be/AQqyGNOP_3o