METE 3100U
Actuators and Power Electronics

Lecture 20

Transient and Dynamics

Outline of Lecture 20

By the end of today's lecture, you should be able to

- Model a separately excited DC generator and motor
- Model the transient of DC generators
- Model the transient of DC motors to disturbances

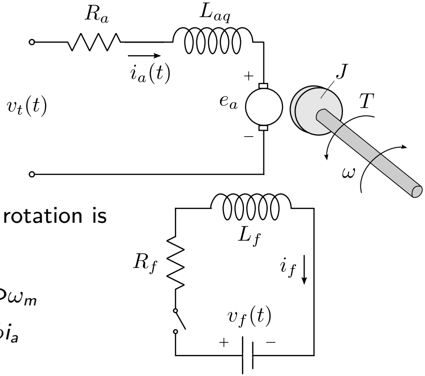
Applications

How does a sudden change in the electrical load affect the resistance torque of the DC generator?

Applications

A sudden change in a synchronous motor load can make it lose synchrony.

DC generator transient



Assumptions:

→ No magnetic saturation

→ No mutual inductance

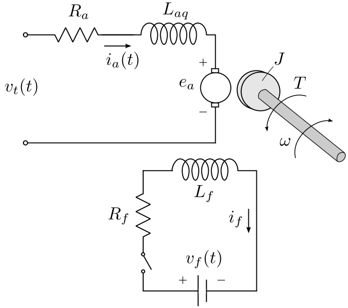
The back emf and torque due to the field rotation is

$$e_a = k_a \Phi \omega_m$$
$$T = k_a \phi i_a$$

If linearity is assumed

$$e_a = k_f i_f \omega_m$$
$$T = k_f i_f i_a$$

Field circuit transient



The machine rotates at a constant speed ω when the switch is closed

The field circuit equations is

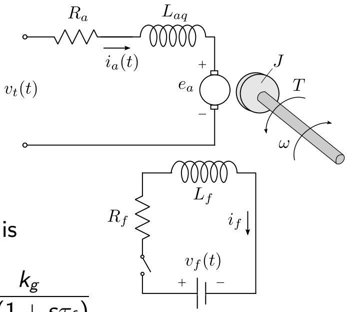
$$V_f = R_f i_r + L_f \frac{di_f}{dt}$$
$$V_f(s) = I_f(s)(R_f + sL_f)$$

The transfer function is

$$\frac{I_f(s)}{V_f(s)} = \frac{1}{L_f s + R_f} = \frac{1}{R_f(1 + s\tau_f)} \qquad (1)$$

where $\tau_f = L_f/R_f$ is the time constant.

Field circuit transient



The generated armature voltage is

$$e_a = k_f i_f \omega_m = k_g i_f$$
$$E_a(s) = k_g I_f(s)$$

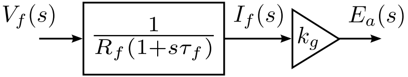
The generated voltage to the field circuit voltage is

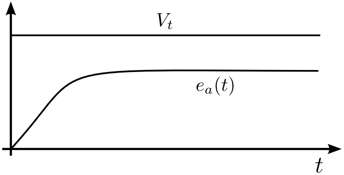
$$\frac{E_a(s)}{V_f(s)} = \frac{E_a(s)}{I_f(s)} = \frac{I_f(s)}{V_f(s)} = \frac{k_g}{R_f(1 + s\tau_f)}$$

In the time domain:

$$e_a(t) = \frac{k_g V_f}{R_f}(1 - e^{-\frac{t}{\tau_f}})$$
$$e_a(t) = E_a(1 - e^{-\frac{t}{\tau_f}})$$

Field circuit transient





$$e_a(t) = \frac{k_g V_f}{R_f}(1 - e^{-\frac{t}{\tau_f}})$$

$$e_a(t) = E_a(1 - e^{-\frac{t}{\tau_f}})$$

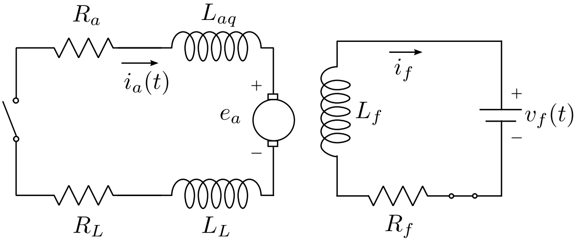
where:

$\Rightarrow E_a = e_a(\infty) = \frac{k_g V_f}{R_f} = \frac{k_g}{I_g}$ is the steady-state generated voltage

$\Rightarrow I_f = \frac{V_f}{R_f}$ is the steady-state current

Armature circuit transient

Line load: R_L and L_L



If the switch is closed when the rotation speed is constant

$$
\begin{aligned}
E_a &= R_a i_a + L_{aq}\frac{di_a}{dt} + R_l L i_a + L_L\frac{di_a}{dt} \\
&= \underbrace{(R_a + R_L)}_{R_{at}} i_a + \underbrace{(L_{aq} + L_L)}_{L_{at}} \frac{di_a}{dt} \\
&= R_{at} i_a + L_{at}\frac{di_a}{dt}
\end{aligned}
$$

Armature circuit transient

The transfer function is

$$E = R_{at} I_a + L_{at} I(s) s \tag{2}$$

$$\frac{I_a(s)}{E_a(s)} = \frac{1}{R_{at}(1 + s\tau_{at})} \tag{3}$$

with $\tau_{at} = L_{at}/R_{at}$ is the armature circuit time constant.

The response to a step change of E_a is

$$i_a(t) = \frac{E_a}{R_{at}} \left(1 - e^{-\frac{t}{\tau_{at}}}\right) \tag{4}$$

The armature current to the **field voltage** is

$$\frac{I_a(s)}{V_f(s)} = \frac{I_a(s)}{E_a(s)} \frac{E_a(s)}{V_f(s)} = \frac{k_g}{R_f R_{at}(1 + s\tau_f)(1 + s\tau_{at})} \tag{5}$$

Field and armature transient

For a step input $V_f(s) = \frac{V_f}{s}$, the current is

$$I_a(s) = \frac{V_f k_g}{R_t R_{at} s(1 + s\tau_f)(1 + s\tau_{at})} \tag{6}$$

Partial fraction expansion yields:

$$I_a(s) = \frac{A_1}{s} + \frac{A_2}{s + 1/\tau_f} + \frac{A_3}{s + 1/\tau_{at}} \tag{7}$$

where

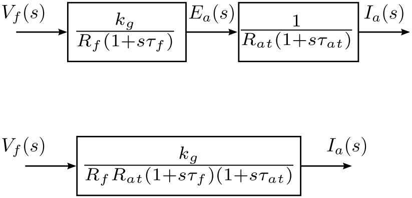
$$A_1 = \left.\frac{A}{(s + 1/\tau_f)(s + 1/\tau_{at})}\right|_{s=0} = A\tau_f\tau_{at} \qquad A = k_g V_f / R_f R_{at} \tau_f \tau_{at}$$

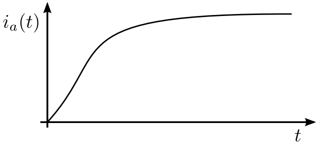
$$A_2 = \left.\frac{A}{s(s + 1/\tau_{at})}\right|_{s=-1/\tau_f} \qquad A_3 = \left.\frac{A}{s(s + 1/\tau_f)}\right|_{s=-1/\tau_{at}}$$

The time domain response is

$$i_a(t) = A_1 + A_2 e^{-\frac{t}{\tau_f}} + A_3 e^{-\frac{t}{\tau_{at}}} \tag{8}$$

Field and armature transient

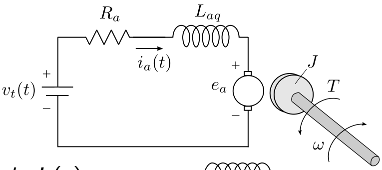




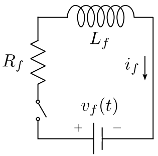
A_1 is the steady-state armature current

$A_1 = i_a\infty = (k_g V_f)/(R_f R_{at}) = k_g I_f / R_{at} = E_a / R_{at}$

Separately-excited DC motor transient



Assuming magnetic linearity:



$$T(s) = k_f I_a(s)(i_f) = k_m I_a(s)$$
$$E_a(s) = k_f \omega(s)(i_f) = k_m \omega(s)$$

where $k_m = i_f k_f$ is a constant.

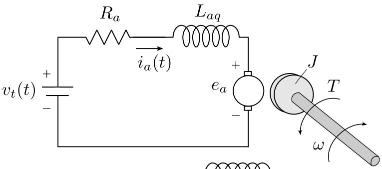
$$V_t(s) = E_a(s) + R_a I_a(s) + L_{aq} s I_a(s)$$
$$V_t(s) = k_m \omega(s) + R_a I_a(s) + L_{aq} s I_a(s)$$
$$V_t(s) = k_m \omega(s) + I_a(s) R_a (1 + s\tau_a)$$

with $\tau_a = L_{aq}/R_a$ is the armature electrical time constant.

Separately-excited DC motor transient

For the mechanical system

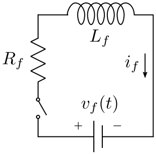
$$T(s) = k_m I_a(s) = Is\omega(s) + b\omega(s) + T_L(s)$$

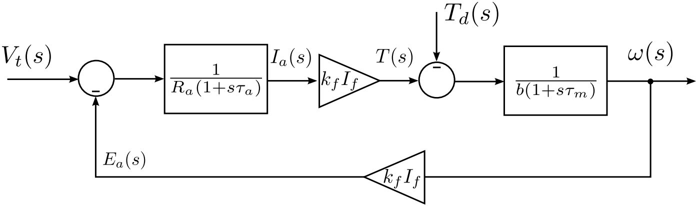


For the electromechanical system

$$\omega(s) = \frac{T(s) - T_L(s)}{b(1 + sJ/b)} = \frac{k_m I_a(s) - T_L(s)}{b(1 + s\tau_m)}$$

$$I_a(s) = \frac{V_t(s) - E_a(s)}{R_a(1 + L_{aq}/R_a)} = \frac{V_t(s) - k_m\omega(s)}{R_a(1 + s\tau_a)}$$





Separately-excited DC motor transient

Scenario 1: Load torque proportional to speed

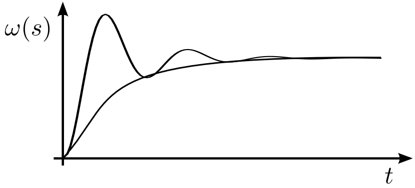
$$T_L(s) = b_L\omega \tag{9}$$

thus

$$T(s) = Js\omega(s) + (b + b_L)\omega(s) \tag{10}$$

The load increases the viscous friction. The transfer function is

$$\frac{\omega(s)}{V_t(s)} = \frac{k_m}{k_m^2 + bR_a(1 + s\tau_m)(1 + s\tau_a)} \tag{11}$$



Separately-excited DC motor transient

Scenario 2: Armature inductance is negligible $\tau_a = 0$

$$\frac{\omega(s)}{V_t(s)} = \frac{k_m}{k_m^2 + bR_a(1 + s\tau_m)} \qquad (12)$$

What is the time response?

Scenario 3: No friction, $b = 0$, with an inertial load J_L

$$T(s) = k_m I_a(s) = (J + J_s)s\omega(s) \qquad (13)$$

thus

$$\frac{\omega(s)}{V_t(s)} = \frac{k_m}{k_m^2 + R_a(J + J_L)s(1 + s\tau_a)} \qquad (14)$$

Separately-excited DC motor transient

Scenario 4: Supply disconnected ($v_t(t) = 0$ at $t = t_1 > 0$ with $\omega(t_1) \neq 0$)

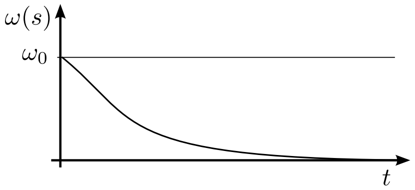
$$T(t) = k_m i_a(t) = Js\omega(t) + b\omega(t) = 0 \quad (15)$$

or $b\omega(t) = -J\frac{d\omega}{dt}$. For an initial speed $\omega(t = t_1) = \omega_0$:

$$b\omega(s) = -J[s\omega(s) - \omega_0] \rightarrow \omega(s) = \frac{\omega_0}{(s + b/J)} = \frac{\omega_0}{s + 1/\tau_m} \quad (16)$$

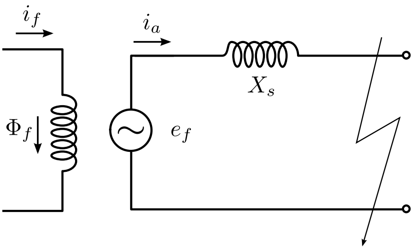
The time domain response is

$$\omega(t) = \omega_0 e^{-\frac{t}{\tau_m}} \quad (17)$$

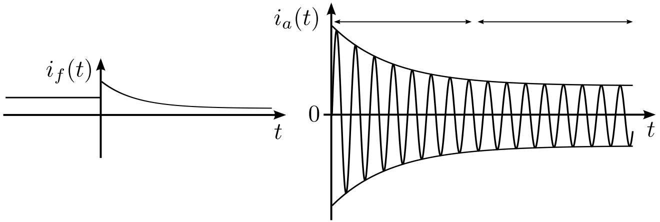


Synchronous machines

The rotor rotates at a constant speed and induces a voltage e_f in the stator

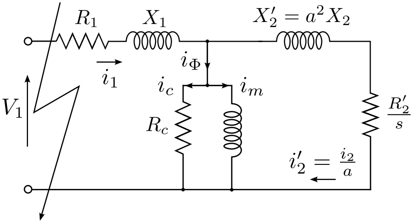


Motor terminals are shorted:

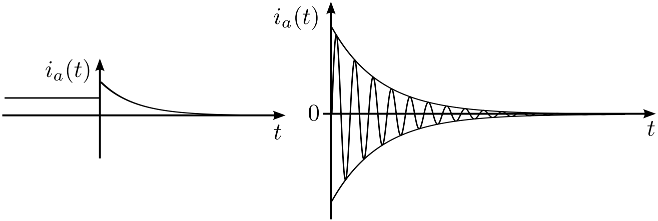


Induction machines

The rotor rotates at a constant speed with a constant current i_1

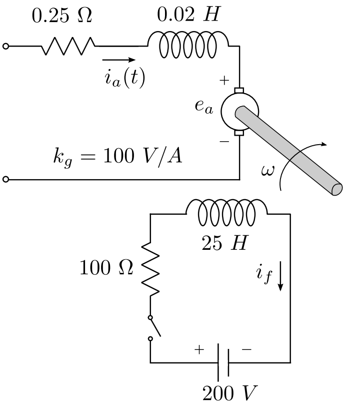


Motor terminals are shorted:



Exercise 92

A separately-exited DC generator is driven at rated speed when a field circuit voltage is suddenly applied to the field winding.



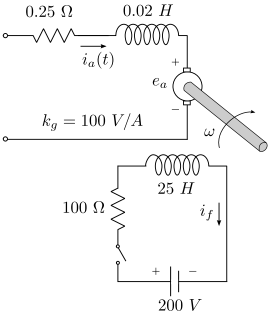
Determine:

(a) The armature-generated voltage as a function of time

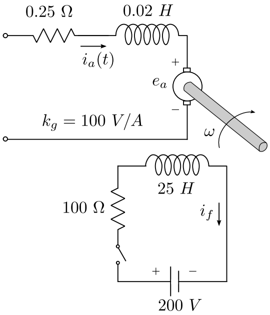
(b) The steady-state armature voltage

(c) The time required to rise the armature voltage to 90% of the steady-state value

Exercise 92 - continued

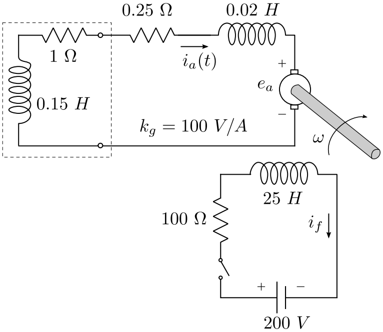


Exercise 92 - continued



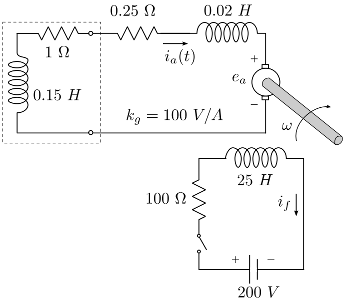
Exercise 93

A separately-exited DC generator is driven at rated speed and a load is connected in series to the generator terminals.



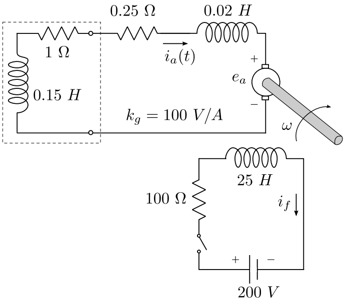
If a field voltage of 200 V is suddenly applied to the field winding, determine the armature current as a function of time.

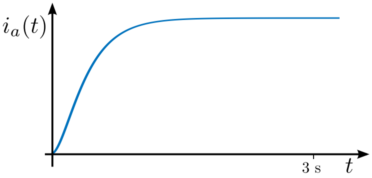
Exercise 93 - continued



Exercise 93 - continued

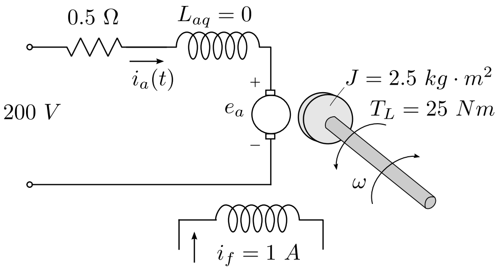
$$i_a(t) = 160 - 351e^{-4t} + 191e^{-7.35t}$$





Exercise 94

The separately exited DC motor generates an open-circuit armature voltage of 220 V when it runs at 2000 rpm with a field current of 1 A.

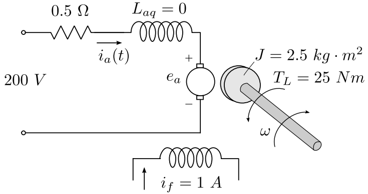


The motor drives a constant load of $T_L = 25\ N \cdot m$. The combined inertia of motor and load is $J = 2.5\ kg \cdot m^2$. The armature terminals are connected to a 220 V DC source and the field current is 1 A.

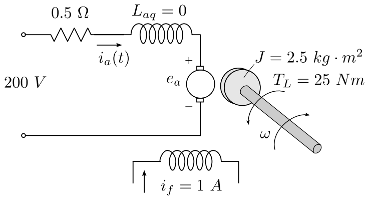
Determine the expressions for the speed and armature current as a function of time, and the steady-state values.

Exercise 94 - continued

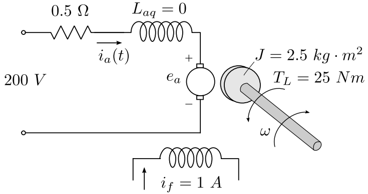
$v_t = 200$ V, no load, 2000 rpm, $i_f = 1$ A.



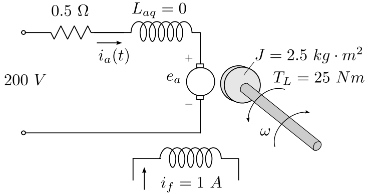
Exercise 94 - continued



Exercise 94 - continued



Exercise 94 - continued



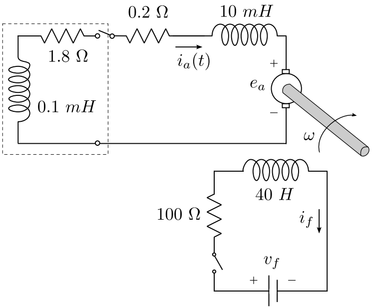
Exercise 95

A separately excited DC generator is driven at 1200 rpm with a field current of 2 A. The armature is then suddenly connected to the load.

(a) Determine the load terminal voltage as a function of time.

(b) Determine the steady-state voltage of the terminal voltage

(c) Determine the torque as a function of time



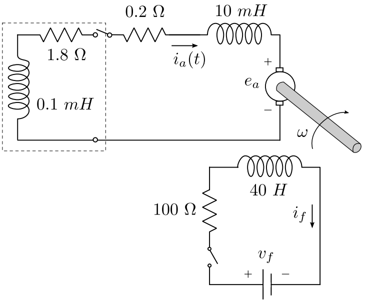
At 1000 rpm, the field constant is $k_g = 100$ V per field ampère.

Exercise 95 - continued

(a) Determine the load terminal voltage as a function of time.

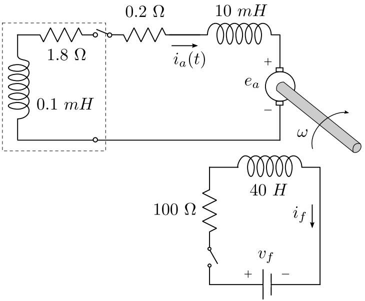
→ 1000 rpm - $k_g = 100$ V/A.

→ 1200 rpm - $k_g = ?$

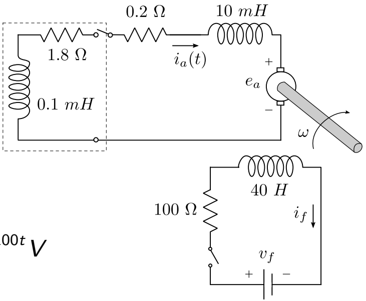


Exercise 95 - continued

(a) The load terminal voltage as a function of time.



Exercise 95 - continued

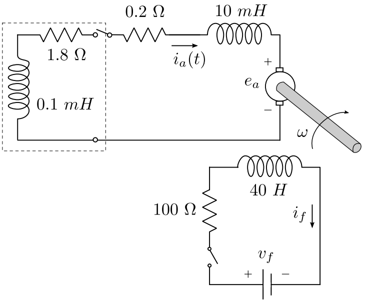


(b) The load terminal steady-state voltage.

$$v_t(t) = 216 - 96e^{-100t}\,V$$

Exercise 95 - continued

(c) The torque as a function of time



Exercise 96

The separately excited DC motor drives a load whose torque is proportional to the speed. The field current is held constant at the rated value when a voltage is suddenly applied across the armature terminals.

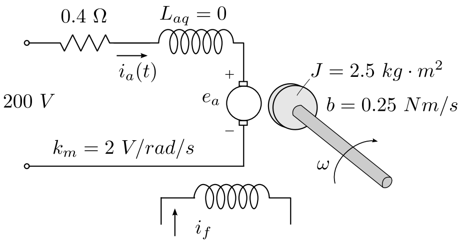
Determine:

(a) The motor speed as a function of time.

(b) The steady-state speed ω_{ss}.

(c) The time required to reach 95% of ω_{ss}.

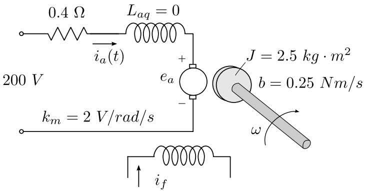
(d) The current $i_a(t)$.



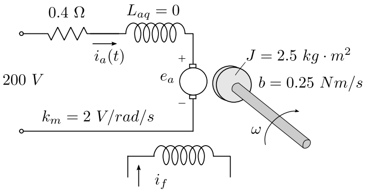
b and J include the motor and load characteristics.

Exercise 96 - continued

(a) The motor speed as a function of time.



Exercise 96 - continued



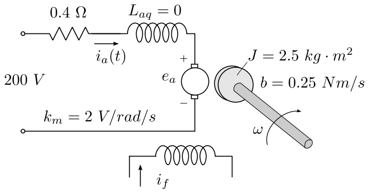
(b) The steady-state speed ω_{ss}.

$$\omega(t) = 97.6(1 - e^{-4.1t})$$

(c) The time required to reach 95% of ω_{ss}.

Exercise 96 - continued

(d) The current $i_a(t)$.



Exercise 97

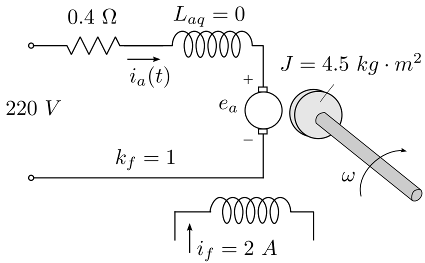
The separately excited DC motor operates at no-load with $v_t = 200$ V and $i_f = 2$ A. Rotational losses are negligible. The motor is intended to be stopped by plugging, that is, by reversal of its armature voltage ($v_t' = -220$ V).

Determine:

(a) The no-load speed.

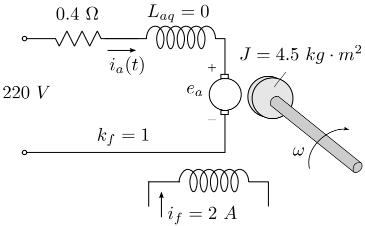
(b) The expression for the speed after plugging.

(c) The time required to stop.



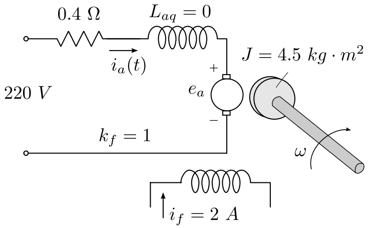
Exercise 97 - continued

(a) The no-load speed.



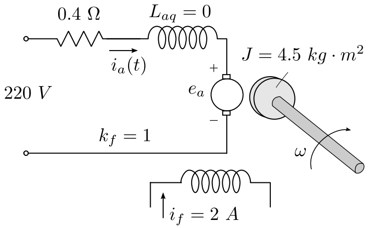
Exercise 97 - continued

(b) The expression for the speed after plugging.



Exercise 97 - continued

(c) The time required to stop.



Next class...

- Final course review