METE 3100U
Actuators and Power Electronics

# Lecture 5
# DC/DC Converters

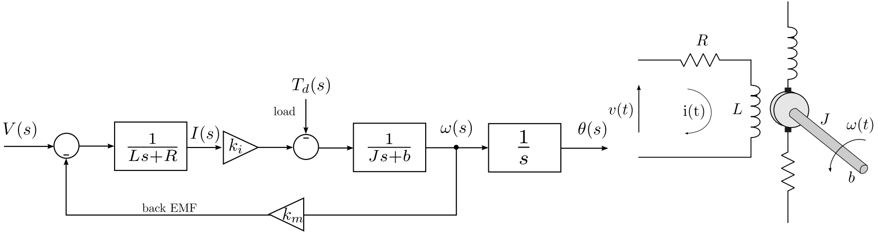
## Outline of Lecture 5

In today's lecture we will

- Understand the principles of DC/DC converters
- Analyse and design DC/DC converters
- Model step-up and step-down converters

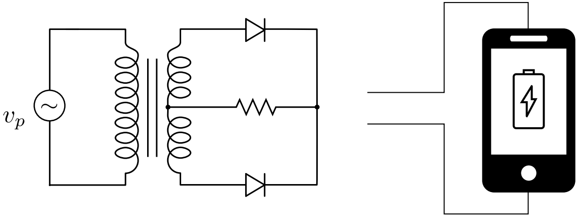
## Applications

Speed control of DC motor requires regulating the voltage applied to the motor. How can a driver apply different voltages to the the motor?



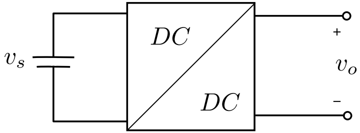
# Applications

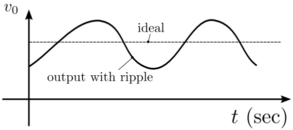
Portable electronic devices such as phones and laptops are supplied with power from batteries. How can the batteries power components that run at different voltages?

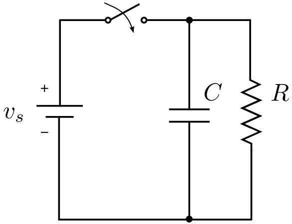


## DC/DC converters

A DC-to-DC circuit or electromechanical converts a source of direct current (DC) from one voltage level to another.

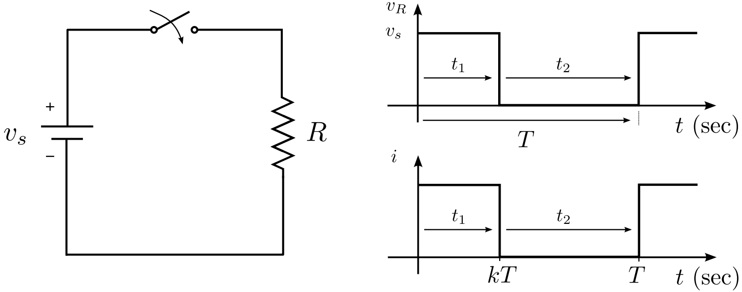


## Step-down operation

Switch opens at $t_1$ remains open for $t_2$ sec. Switch closes again at $T = t_1 + t_2$.



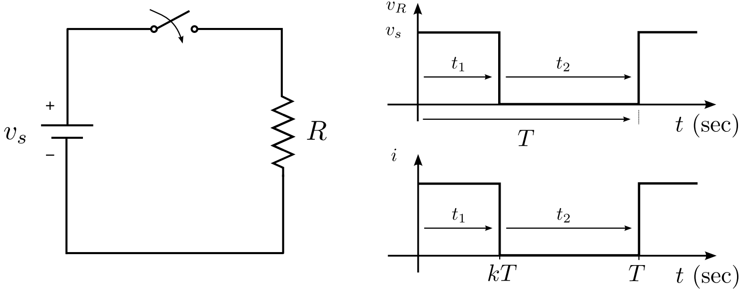

The average voltage across the resistor is

$$V_R = \frac{1}{T}\int_0^{t_1} v_s dt = \frac{t_1}{T} v_s = k v_s = (\omega t_1) v_s \tag{1}$$

$k = \frac{t_1}{T}$ is called the duty cycle with $0 \leq k \leq 1$

## Step-down operation

Switch opens at $t_1$ remains open for $t_2$ sec. Switch closes again at $T = t_1 + t_2$.




The rms voltage across the resistor is

$$V_0 = \sqrt{\frac{1}{T}\int_0^{kT} v_R^2 dt} = \sqrt{k}v_s \tag{2}$$

$k = \frac{t_1}{T}$ is called the duty cycle with $0 \leq k \leq 1$

## Step-down operation

Assuming a lossless converter, the power is

$$P_i = \frac{1}{T}\int_0^{kT} v_0 i dt = \frac{1}{T}\int_0^{kT} \frac{v_0^2}{R} dt = k\frac{v_s^2}{R} \tag{3}$$

The effective input resistance seen by the source is

$$R_i = \frac{v_s}{i_{av}} = \frac{v_s}{kv_s/R} = \frac{R}{k} \tag{4}$$

This indicates that the converter makes the input resistance as a variable resistance of $R/k$.

In summary:

→ The duty-cycle can be varied from 0 to 1

→ The output voltage can vary from 0 to $v_s$

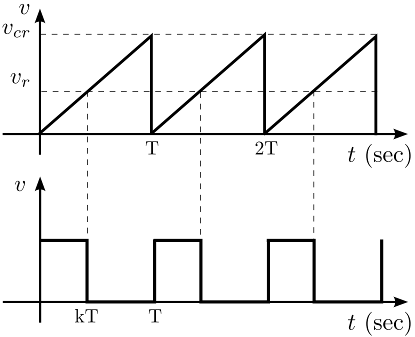
→ The output power is a function of $k$

## Duty cycle generation

→ Generate a triangular waveform $v_{cr}$ of period $T$

→ Compare $v_{cr}$ with a DC carrier signal $v_r$

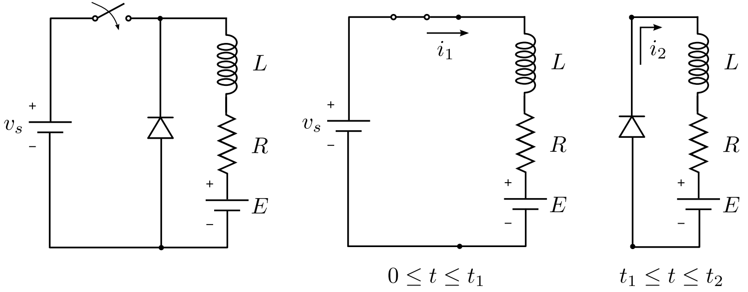
→ Any variation of $v_r$ will change $k$ linearly



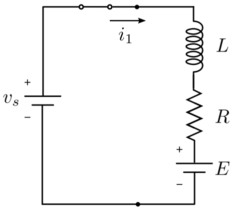
## Step-down with RL load

Switch is closed from $0 \leq t \leq t_1$: Current $i_1$ rises

Switch is opened ($t = t_2$): Current $i_2$ decays



## Step-down with RL load




Switch is closed from $0 \leq t \leq t_1$:

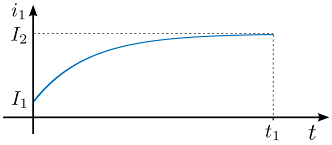
$$v_s = Ri_1 + L\frac{di_1}{dt} + E$$

If $i(0) = I_1$, the load current is

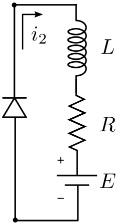
$$i_1(t) = I_1 e^{-t\frac{R}{L}} + \frac{v_s - E}{R}\left(1 - e^{-t\frac{R}{L}}\right) \tag{5}$$

When the switch is closed at $t = t_1$, the current is

$$I_2 = i_1(t_1) \tag{6}$$



## Step-down with RL load




Switch is opened at $t = t_1$, the initial current is $I_2$

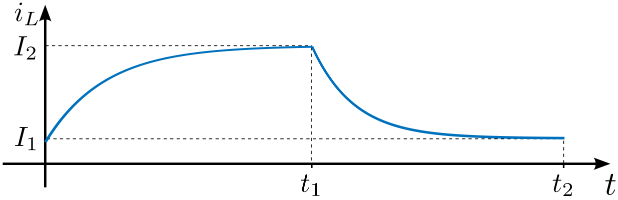
Mode 2 $\rightarrow$ Time is reset to $t = 0$

$$0 = Ri_2 + L\frac{di_2}{dt} + E$$

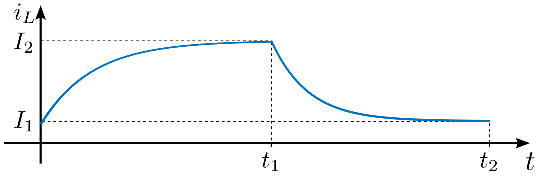
If $i(0) = I_2$, the load current is

$$i_2(t) = I_2 e^{-t\frac{R}{L}} - \frac{E}{R}\left(1 - e^{-t\frac{R}{L}}\right) \tag{7}$$

At the end of this mode $i_2(t_2) = I_3$. Under steady-state $I_3 = I_1$.



## Step-down with RL load




Combining the previous equations gives:

$$I_2 = I_1 e^{-\frac{kTR}{L}} + \frac{V_s - E}{R}\left(1 - e^{-\frac{-kTR}{L}}\right)$$

$$I_1 = I_2 e^{-\frac{(1-k)TR}{L}} - \frac{E}{R}\left(1 - e^{-\frac{(1-k)TR}{L}}\right)$$

Setting $z = \frac{TR}{L}$ and solving for $I_1$ and $I_2$ yields

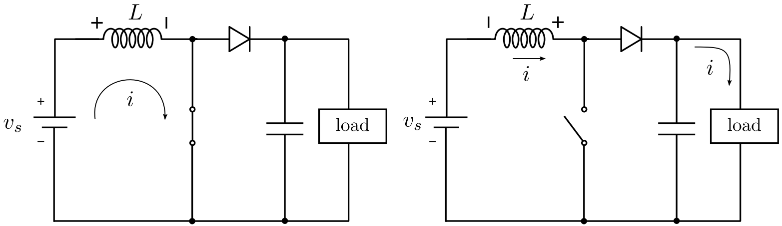
$$I_1 = \frac{V_s}{R}\left(\frac{e^{kz} - 1}{e^{z} - 1}\right) - \frac{E}{R}, \qquad I_2 = \frac{V_s}{R}\left(\frac{e^{-kz} - 1}{e^{-z} - 1}\right) - \frac{E}{R}$$

The peak-to-peak ripple current is $\Delta i = I_2 - I_1$

## Step-up converters

**Switch is closed**: Energy is stored in the inductor

**Switch is opened**: Sudden drop in current causes the inductor to produce a back *emf* in the opposite polarity to the voltage across it.

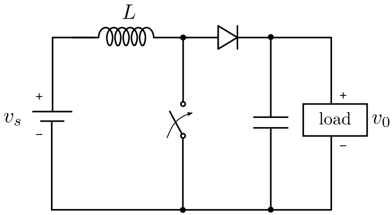


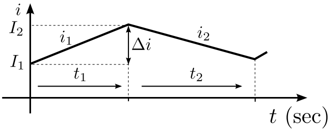

The voltage across the load becomes $v_s + v_L$

## Step-up converters

Switch is closed ($t_1$): Inductor current rises ($i_1$)

Switch is opened ($t_2$): Inductor current goes to the load ($i_2$)



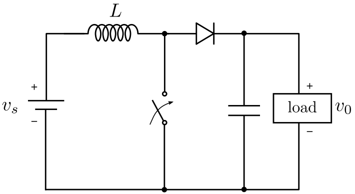

When the switch is closed, the voltage across the induction is

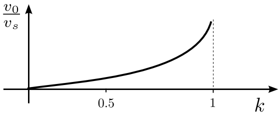
$$v_s = L\frac{di}{dt} \approx L\frac{\Delta i}{t_1} \tag{8}$$

which gives a peak-to-peak current as

$$\Delta i = \frac{v_s}{L} t_1 \tag{9}$$

## Step-up operation




When the switch is opened, the output voltage is

$$v_0 = v_s + L\frac{\Delta i}{t_2} \tag{10}$$

since $\Delta_i = v_s t_1/L$:

$$v_0 = v_s + v_s\frac{t_1}{t_2} = v_s\left(1 + \frac{t_1}{t_2}\right) \tag{11}$$

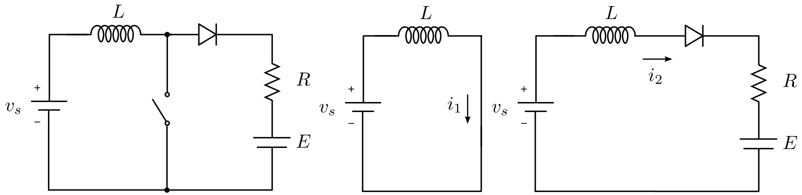
Since $t_1 + t_2 = T$ and the duty cycle is $k = t_1/T$:

$$v_0 = v_s\left(\frac{t_1 + t_2}{t_2}\right) = v_s\frac{T}{T - t_1}, \quad \Rightarrow v_0 = v_s\frac{1}{1-k} \tag{12}$$

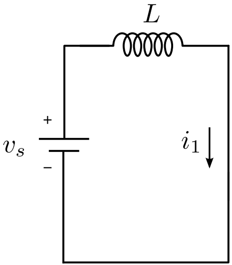
## Step up converter with a resistive load

Switch is closed from $0 \leq t < t_1$: Current $i_1$ loads the inductor

Switch is opened ($t = t_1$): Current $i_2$ flows to the load



## Step up converter with a resistive load




Switch is closed from $0 \leq t \leq t_1$:

$$v_s = L\frac{di_1}{dt}$$

For an ideal circuit ($R = 0$) with an initial current $I_1$:

$$i_1(t) = \frac{v_s}{L}t + I_1$$

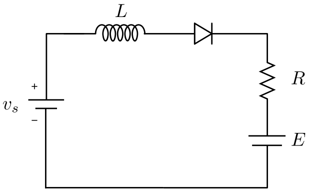
During mode 1, the current must rise. The the necessary condition is

$$\frac{di_1}{dt} > 0, \quad \text{or} \quad v_s > 0 \tag{13}$$

This is valid for $0 \leq t \leq kT$. At $t = t_1 = kT$, the current is


$$I_2 = i_1(t_1) = \frac{v_s}{L}kT + I_1 \tag{14}$$

## Step up converter with a resistive load




Switch is opened at $t = t_2$:

$$v_s = Ri_2 + L\frac{di_2}{dt} + E$$

With an initial current $I_2$, the current is

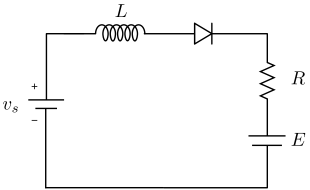
$$i_2(t) = \frac{v_s - E}{R}\left(1 - e^{-t\frac{R}{L}}\right) + I_2 e^{-t\frac{R}{L}}$$

$\forall\ t_1 \leq t \leq (1-k)T$. When $t = t_2 = (1-k)T$, the current is

$$I_1 = i_2(t_2) = \frac{v_s - E}{R}\left[1 - e^{-(1-k)\frac{TR}{L}}\right] + I_2 e^{-(1-k)\frac{TR}{L}} \qquad (15)$$

What is $I_1$ ?

## Step up converter with a resistive load




$$I_1 = \frac{v_s - E}{R}\left[1 - e^{-(1-k)\frac{TR}{L}}\right] + I_2 e^{-(1-k)\frac{TR}{L}}$$

$$I_2 = \frac{v_s}{L}kT + I_1$$

Solving for $I_1$ and $I_2$:

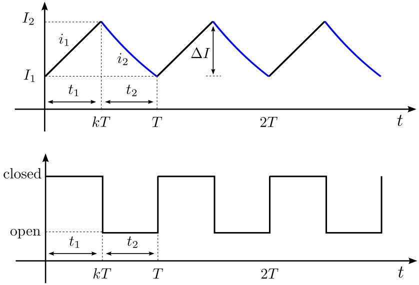
$$I_1 = \frac{v_s kT}{L}\frac{z}{1-z} + \frac{v_s - E}{R}\ , \qquad I_2 = \frac{v_s kT}{L}\frac{1}{1-z} + \frac{v_s - E}{R}$$

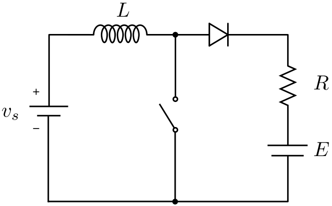
with $z = e^{-(1-k)\frac{TR}{L}}$. The ripple current is

$$\Delta I = I_2 - I_1 = \frac{v_s kT}{L}\left(\frac{1}{1-z} - \frac{z}{1-z}\right)$$

$$\Delta I = \frac{v_s kT}{L}$$

## Step up converter with a resistive load

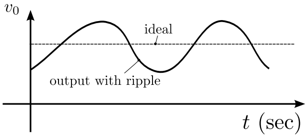



→ Use the Matlab code to analyse the influence of $L$ and $k$

→ Verify the results with the LTspice circuit

## Measures of performance




The DC output power is

$$P_{dc} = I_a V_a$$

where $V_a$ and $I_a$ are the average load voltage and current.

The AC output power is

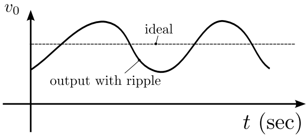
$$P_{ac} = I_0 V_0 \tag{16}$$

where $V_0$ and $I_0$ are the rms load voltage and current.

The converter efficiency is

$$\mu = \frac{P_{dc}}{P_{ac}} \tag{17}$$

## Measures of performance




The rms ripple content of the output voltage is

$$V_r = \sqrt{V_0^2 - V_a^2}$$

The rms ripple content of the input current is

$$I_r = \sqrt{I_0^2 - I_a^2} \tag{18}$$

where $I_0$ and $I_a$ are the rms and average dc supply current. The ripple factor of output voltage is

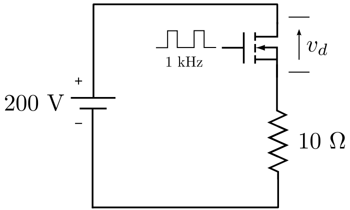
$$RF_v = \frac{V_r}{V_a} \tag{19}$$

The ripple factor of the input current is

$$RF_i = \frac{I_r}{I_a} \tag{20}$$

## Exercise 15

The converter uses a Mosfet transistor as a switch. When the switch is on, its voltage drops by 2V. If the chopping frequency is 1 kHz and the duty cycle is 50%, determine:




**(a)** The average output voltage

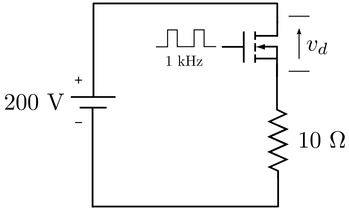
**(b)** The rms output voltage

**(c)** The converter efficiency

**(d)** The effective input resistance

**(e)** The ripple factor of the output voltage

## Exercise 15 - continued




$v_d = 2$ V, $f = 1$ kHz, $k = 50\%$.

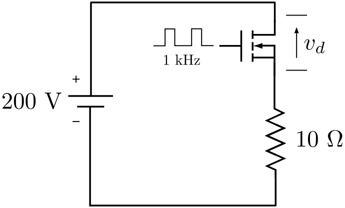
**(a)** The average output voltage

**(b)** The rms output voltage

# Exercise 15 - continued

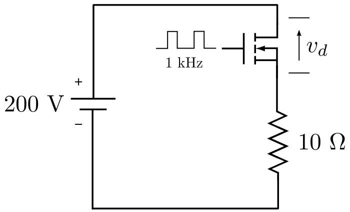
$v_d = 2$ V, $f = 1$ kHz, $k = 50\%$.

**(c)** The converter efficiency



## Exercise 15 - continued

$v_d = 2$ V, $f = 1$ kHz, $k = 50\%$.




**(d)** The effective input resistance

**(d)** The ripple factor of the output voltage

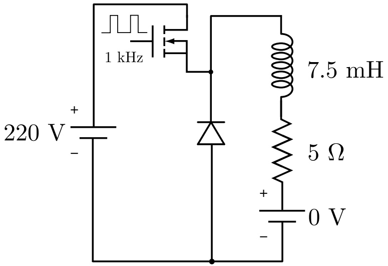
## Exercise 16

The DC/DC converter uses a Mosfet transistor as a switch. If the chopping frequency is 1 kHz and the duty cycle is 50%, determine:

**(a)** The peak to peak ripple load current

**(b)** The average load current

**(c)** The rms load current



## Exercise 16 - continued

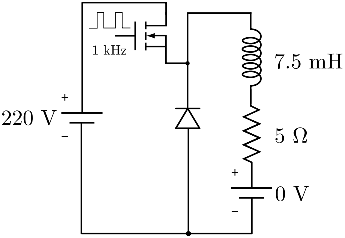
$f = 1$ kHz, $k = 50\%$.

**(a)** The peak to peak current

$$I_1 = \frac{V_s}{R}\left(\frac{e^{kz}-1}{e^{z}-1}\right) - \frac{E}{R},$$

$$I_2 = \frac{V_s}{R}\left(\frac{e^{-kz}-1}{e^{-z}-1}\right) - \frac{E}{R},$$

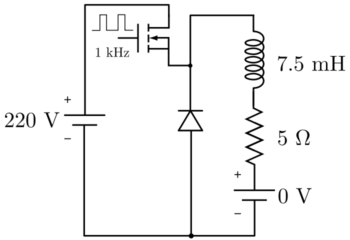
$$z = \frac{TR}{L}$$



## Exercise 16 - continued

$f = 1$ kHz, $k = 50\%$.

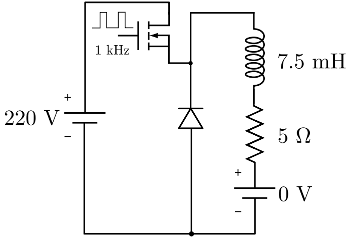
**(b)** The average load current



# Exercise 16 - continued

$f = 1$ kHz, $k = 50\%$.

**(c)** The rms load current



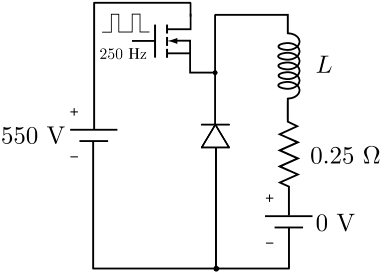
## Exercise 17 - Design question

For the design project. Otherwise skip this question.

The average load current in the converter is 200 A. If the chopping frequency is 250 Hz, using the average output voltage calculate the load inductance $L$ required to limit the maximum load ripple current to 10% of its average.

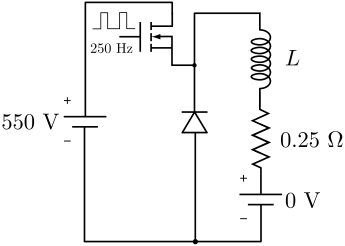
**Procedure**:

- Calculate the voltage across the inductor
- Assume the load current to rise linearly
- Calculate $k$ for the maximum ripple current
- Calculate the inductance

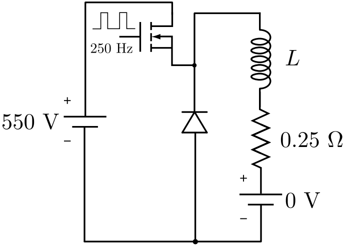


# Exercise 17 - continued

$\Delta I < 0.1 I_a$

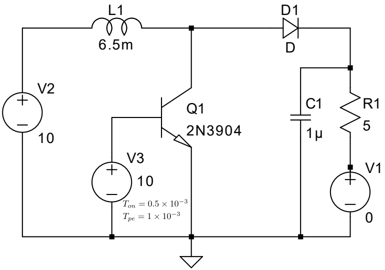


# Exercise 17 - continued



## Exercise 18

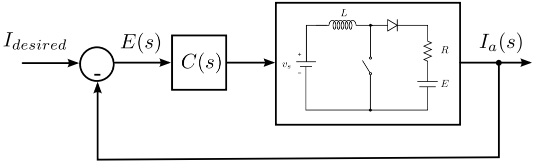
The step-up converter has an input voltage of 10 V, a chopping frequency of 1 kHz and a duty cycle of 50%. Create a SPIC simulation follow the schematic to find the values of diode, switch, and load current.




A LTspice file is posted on BlackBoard.

# Exercise 19

DC/DC converters can be used in a feedback control system to regulate the speed or torque delivered by a DC motor. The duty-cycle is controlled to provide a constant voltage or current, respectively. Draw the schematic diagram of a rudimentary block scheme of a control structure.




Install the Simscape Matlab extension

Matlab command: pe_boost_converter_control

# Exercise 19 - continued

# Next class...

- Pulse width modulation

Additional supporting materials for Lecture 5:

Step-up converters: `https://goo.gl/8ZYHV3`

Step-up converter for current control: `https://bit.ly/2UFkaAU`

Step-up converter for voltage control: `https://bit.ly/2EuruKY`