METE 3100U
Actuators and Power Electronics

# Lecture 9

# Fundamentals of Electromechanical Energy Conversion

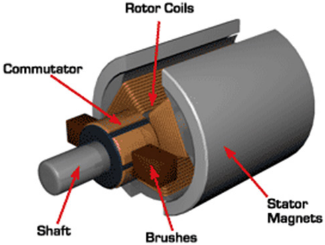
## Outline of Lecture 9

By the end of today's lecture, you should be able to

- Calculate the stored electromagnetic energy
- Understand the principles of energy conversion
- Model and analyse an electromagnet

# Applications

DC motors convert electrical energy into mechanical energy and vice-versa.
How is the energy converted?



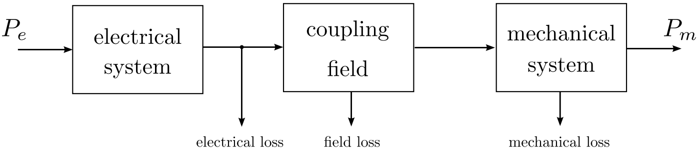
# Applications

The magnetic levitation train uses two sets of magnets, one set to repel and push the train up off the track. How can them be used to propel the train?

# Applications

Electromagnets and solenoids are widely used in mechatronic systems. How can we calculate the force deployed by these actuators?

## Energy conversion process




Principle of conservation of energy

$$W_e = W_m + W_f + W_{loss} \tag{1}$$

→ $W_e$: electrical energy

→ $W_m$: mechanical energy

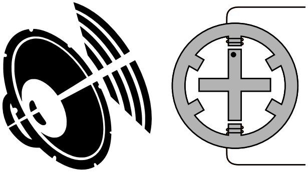
→ $W_f$: field stored energy

→ $W_{loss}$: all energy losses

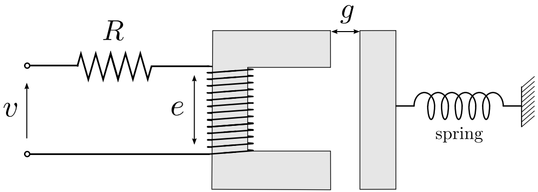
  → **Electrical**: Joule's loss $i^2R$

  → **Mechanical**: Friction $b$

  → **Magnetic**: Eddy currents



## Energy conversion process




Neglecting losses, the instantaneous energy change during a time interval $dt$ is

$$dW_e = dW_m + dW_f + W_{loss} \tag{2}$$

If $\dot{g} = 0$, then $W_m = 0$ and thus

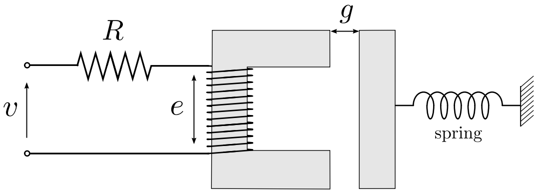
$$dW_e = dW_f \tag{3}$$

The electric energy is

$$dW_e = eidt \tag{4}$$

**What is the field energy?**

## Energy conversion process




The differential field energy is

$$dW_f = id\lambda \tag{5}$$

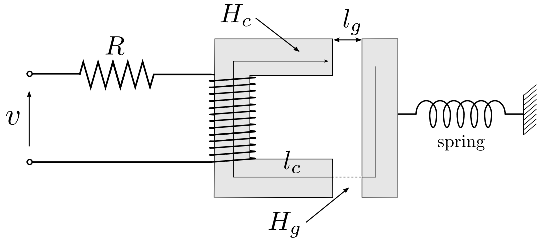
Integrating both sides yields

$$W_f = \int_0^{\lambda} id\lambda \tag{6}$$

where $\lambda$ is the coil flux linkage defined by

$$\lambda = \int edt \rightarrow e = \frac{d\lambda}{dt} \tag{7}$$

## Energy conversion process




The electromotive force relates to the magnetic field intensity $H$ as (Lecture 2):

$$Ni = H_c \ell_c + H_g \ell_g \rightarrow i = \frac{H_c \ell_c + H_g \ell_g}{N} \tag{8}$$

Thus

$$W_f = \int_0^{\lambda} \frac{H_c \ell_c + H_g \ell_g}{N} d\lambda \tag{9}$$

($\ell$ is the the length of a magnetic path)

## Field energy

According to Faraday's law (Lecture 2):

$$\Phi = \int_A \vec{B} \cdot d\vec{A} \tag{10}$$

$A$ is the cross-sectional area. For a coil with $N$ turns and no leakage:

$$\lambda = N\Phi = NBA \rightarrow d\lambda = NAdB \tag{11}$$

which yields:

$$W_f = \int \frac{H_c \ell_c + H_g \ell_g}{N} NAdB \tag{12}$$

If the flux density $B = \phi/A$ is constant, for the air gap we have

$$H_g = \frac{B}{\mu_0} \tag{13}$$

$$W_f = \int H_c \underbrace{A\ell_c} dB + \int \frac{B}{\mu_0} \underbrace{A\ell_g} dB \tag{14}$$

## Field energy

$$W_f = \int H_c \underbrace{A\ell_c}_{V_c} dB + \int \frac{B}{\mu_0} \underbrace{A\ell_g}_{V_g} dB$$

$$W_f = \left( \int H_c dB \right) V_c + \left( \frac{B^2}{2} \frac{1}{\mu_0} \right) V_g$$

$$W_f = W_{fc} + W_{fg}$$

→ $w_{fc} = \int H_c dB_c$ is the energy **density** in the magnetic core

→ $w_{fg} = \frac{B^2}{2\mu_0}$ is the energy **density** in the air gap

→ $V_g,\ V_c$, are the volume of the air gap and magnetic core

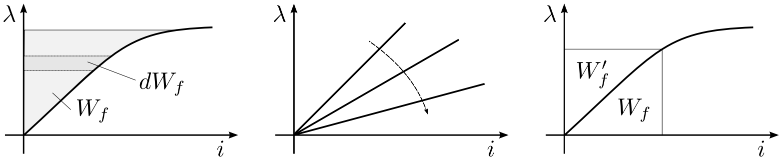
→ $W_{fc}$ is the energy in the magnetic core

→ $W_{fg}$ is the energy store in the air gap

**Typically**: $W_{fg} >> W_{fc}$ and thus

$$W_f \approx W_{fg} \tag{15}$$

## Energy and coenergy




$$W_f = \int_0^{\lambda} i d\lambda, \qquad W_f = \int_0^{i} \lambda di \tag{16}$$

For a linear system

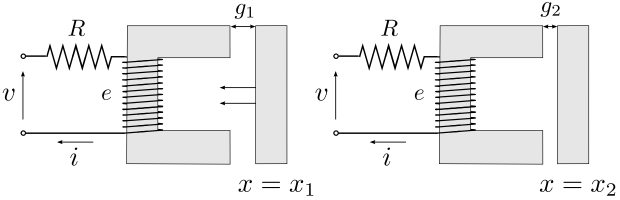
$$W_f' = W_f \tag{17}$$

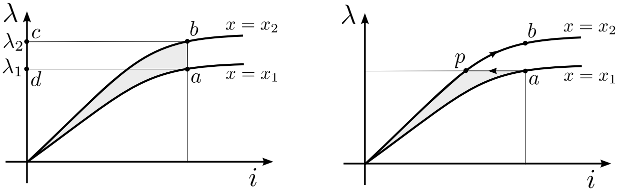
→ $W_f$ if the field energy

→ $W_f'$ if the field coenergy

## Mechanical force

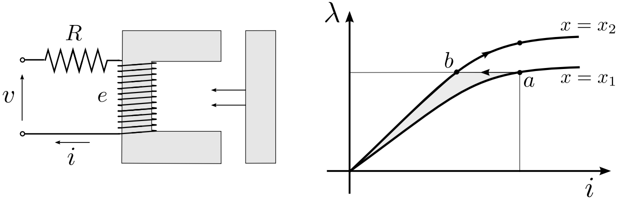
The core moves from $x = x_1$ (point $a$) to $x = x_2$ (point $b$), with $g_2 < g_1$



## Mechanical force

**Case 1**: The movement occurs quickly. What happens to the current?




If $\lambda$ is constant, the work done is a decrease in the field energy

$$dW_m = dW_f \tag{18}$$

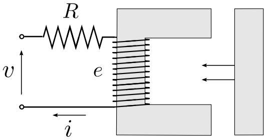
If $f_m$ is the mechanical force causing a displacement $dx$

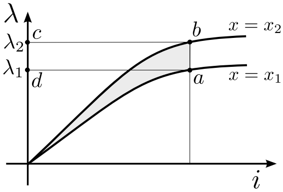
$$f_m dx = dW_f$$

$$f_m = \left.\frac{\partial W_f(\lambda, x)}{\partial x}\right|_{\lambda = cte}$$

## Mechanical force

**Case 2**: The movement occurs slowly. What happens to the current?




If $i$ is constant, the work done is an decrease in the field co-energy

$$dW_m = dW'_f \tag{19}$$

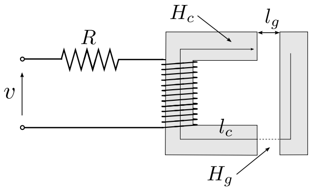
If $f_m$ is the mechanical force causing a displacement $dx$

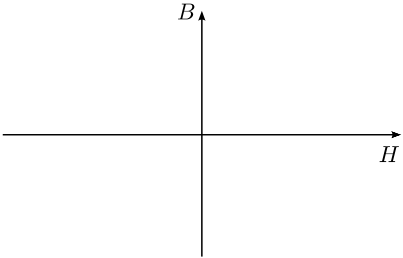
$$f_m dx = dW'_f$$

$$f_m = \left. \frac{\partial W'_f(i,x)}{\partial x} \right|_{i=cte}$$

## Experiment - Hysteresis

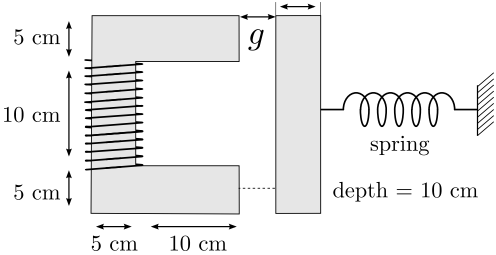
An external magnetic field causes the atomic dipoles to align themselves with it



## Exercise 38

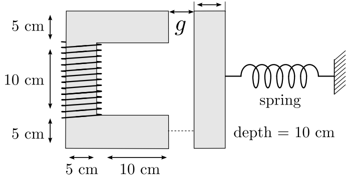
The magnetic core of the actuator shown is made of cast steel with a magnetic permeability of $\mu = 1.5 \times 10^{-3}$ Hm$^{-1}$. The coil has 250 turns and the coil resistance is 5 Ω. For a fixed air gap of 5 mm, a DC supply is connected to the coil to produce a flux density of 1 Tesla in the air gap.



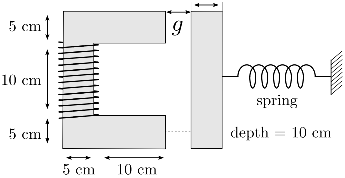

**(a)** Calculate the required DC voltage

**(b)** Calculate the store field energy

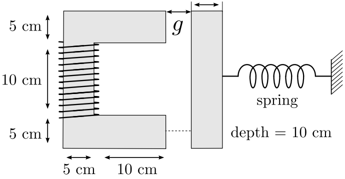
# Exercise 38 - continued



# Exercise 38 - continued



## Exercise 38 - continued



# Exercise 39

The relationship between the magnetic flux linkage $\lambda$ and the current $i$ of an electromagnet is given by

$$i = \left(\frac{\lambda}{0.09} g\right)^2$$

which is valid for the limits $0 < i \leq 4\ A$ and $3 < g < 10$ cm. For a current of 3 Amps and air gap length of $g = 5$ cm, find the mechanical force on the moving part using the energy and co-energy of the field.

# Exercise 39 - continued

# Exercise 39 - continued

## Exercise 40

In a translational motion actuator, the $\lambda - i$ relationship is given by

$$i = \lambda^{\frac{3}{2}} + 2.5\lambda(x-1)^2 \quad (20)$$

for $0 < x < 1$ m, where $i$ is the current in the coil of the actuator. Determine the force on the moving part at $x = 0.6$ m.

# Exercise 40 - continued

# Exercise 40 - continued

## Exercise 41

In a translational motion actuator, the $\lambda - i$ relationship is given by

$$\lambda = 1.2\frac{i^{\frac{1}{2}}}{g} \tag{21}$$

where $g$ is the air gap length. For a current $i = 2$ A and when $g = 10$ cm, determine the mechanical force on the moving part using

**(a)** The stored field **energy**

**(b)** The stored field **coenergy**

# Exercise 41 - continued

# Exercise 41 - continued

## Next class...

- Mechanical force in electromagnetic systems

Additional supporting materials for Lecture 9:

What is coenergy?: `https://youtu.be/dDTnoQTeA24`

Example of an induction motor: `https://youtu.be/FHgw_l-Z_s0`