MECE 3350U
Control Systems

Lecture 2

Dynamic Models

Videos in this lecture

Lecture 2: https://youtu.be/qDG0nENYpDg

Exercise 3: https://youtu.be/npGb3FwYQjk

Exercise 4: https://youtu.be/-fwBwwGMZRY

Exercise 5: https://youtu.be/N_hq63DgUY0

Exercise 6: https://youtu.be/_21rf1t3H80

Exercise 7: https://youtu.be/eXgsnemhjvs

Exercise 8: https://youtu.be/e2Xq_gh8Rs4

Exercise 9: https://youtu.be/FAcFXZyPGdU

Exercise 10: https://youtu.be/lhNh_4koB8s

Outline of Lecture 2

By the end of this lecture you should be able to

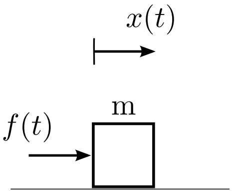
- Model mechanical and electrical systems
- Find the differential equation that describes the behaviour of a physical system
- Understand the analogy between mechanical and electrical systems

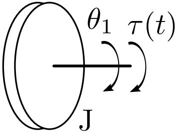
Elements of a mechanical system

→ **Mass**: The quantity of matter in a body

→ **Inertia**: Tendency to resist changes in state of motion

Idealization: Rigid body

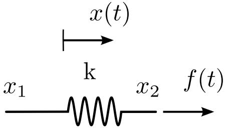


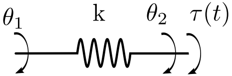


Elements of a mechanical system

→ **Spring**: Designed to store energy

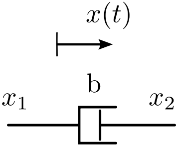
Idealization: Negligible mass and damping



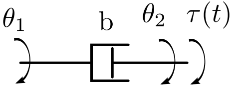


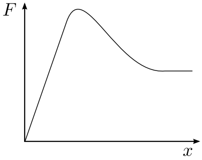
Elements of a mechanical system

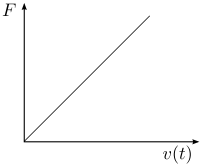
→ **Viscous damper**: Designed to dissipate energy



Idealization: negligible mass and stiffness



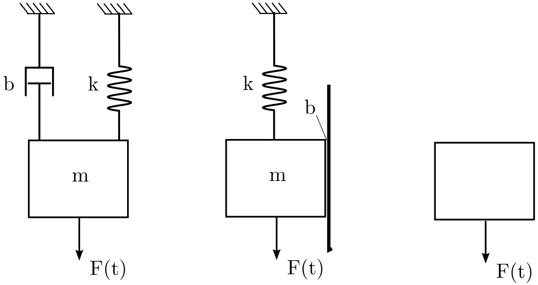




viscous friction
$\neq$
kinetic/static friction

Example

Find the equation of motion of the spring-mass-damper system.



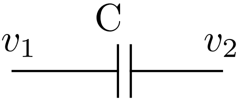
Elements of electrical circuits

→ **Resistor**: Resistance against electric current

Idealization: No inductance or capacitance

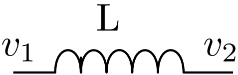
→ **Capacitor**: Stores energy in an electric field

Idealization: No inductance or resistance



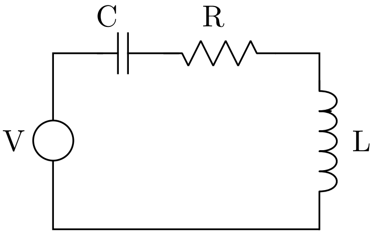
→ **Inductor**: Stores energy in a magnetic field

Idealization: No capacitance or resistance

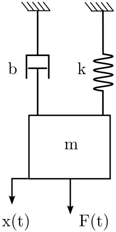


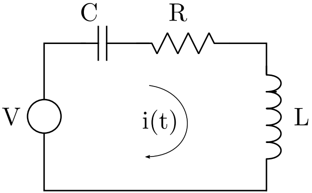
Example

Find the relation between the voltage V, the current, and the charge in the circuit.



Mechanical/electrical analogy





$$F = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx \tag{1}$$

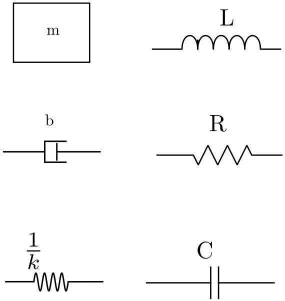
$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q \tag{2}$$

Impulse response

$$x(t) = Ke^{-\alpha t}\sin(\beta t + \theta) \tag{3}$$

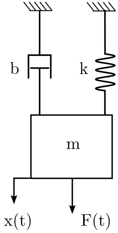
Mechanical/electrical analogy

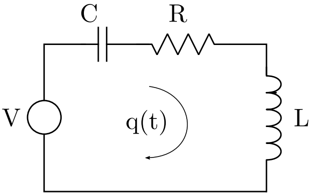
Mechanical		Electrical	
Force	F	Voltage	V
Velocity	v	Current	i
Displacement	x	Charge	q
Damping	b	Resistance	R
Mass	m	Inductance	L
Compliance	k^{-1}	Capacitance	C

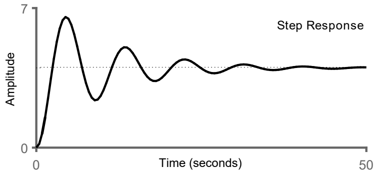


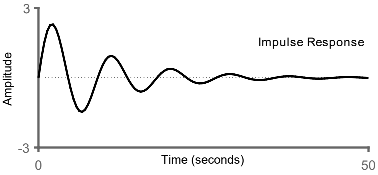
Mechanical/electrical analogy

Taking: $m = 5$ kg, $k = 0.25$ N/m, $b = 0.1$ Ns.









Mechanical/electrical analogy

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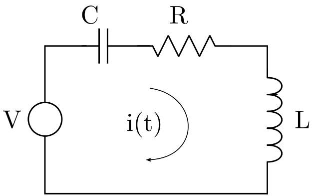
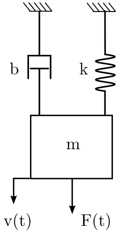
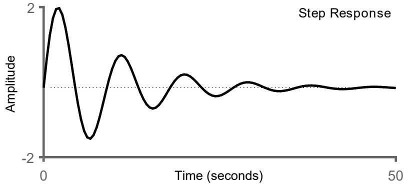
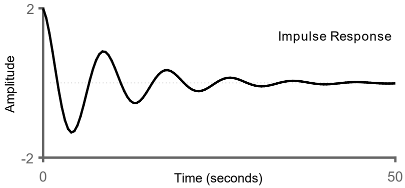
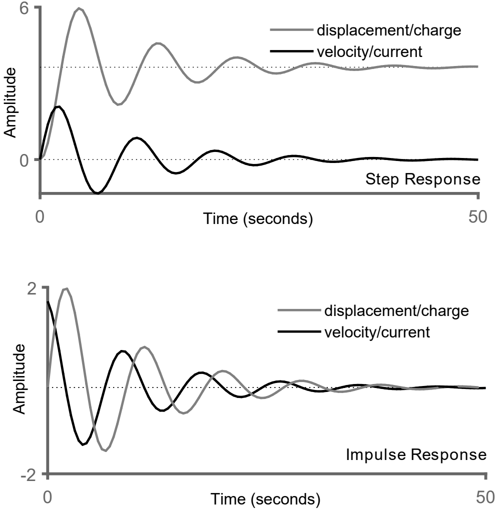






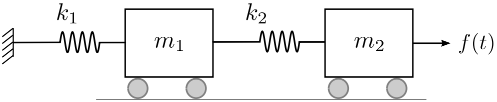


Mechanical/electrical analogy



Exercise 3

Find the equations of motion of the mass-spring system shown.

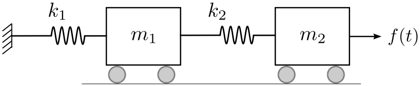


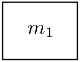
Procedure:

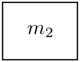
$\rightarrow$ Draw the free body diagram of each mass

$\rightarrow$ Apply the equation of motion

Exercise 3 - continued

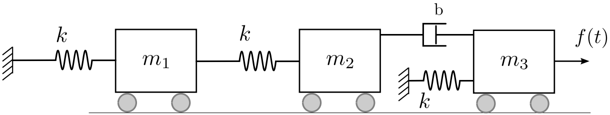






Exercise 4

Find the differential equations to model the behaviour of the system shown.

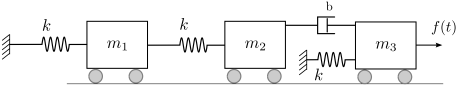


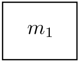
Procedure:

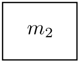
→ Draw the free body diagram of each mass

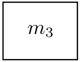
→ Apply the equation of motion

Exercise 4 - continued



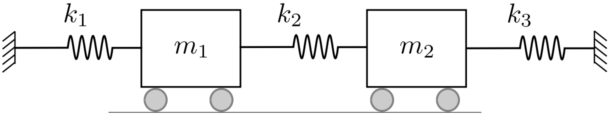






Exercise 5

Find the differential equations to model the behaviour of the system shown.

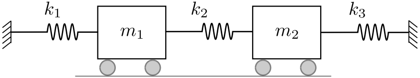


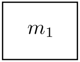
Procedure:

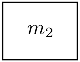
→ Draw the free body diagram of each mass

→ Apply the equation of motion

Exercise 5 - continued

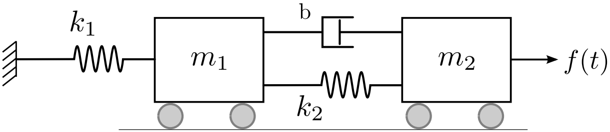






Exercise 6

Find the differential equations to model the behaviour of the system shown.

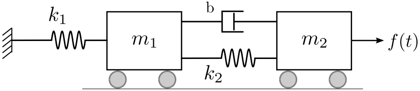


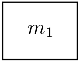
Procedure:

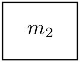
→ Draw the free body diagram of each mass

→ Apply the equation of motion

Exercise 6 - continued

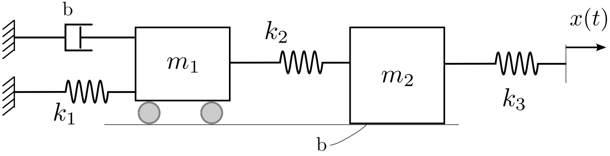






Exercise 7

Find the differential equations to model the behaviour of the system shown.

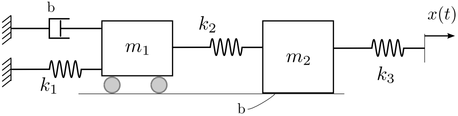


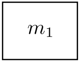
Procedure:

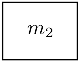
→ Draw the free body diagram of each mass

→ Apply the equation of motion

Exercise 7 - continued

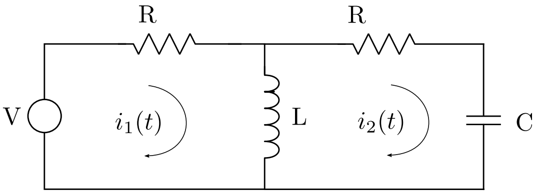






Exercise 8

Write the the differential equations ($i = f(V)$) of the following circuit.

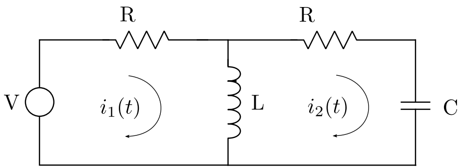


Procedure:

→ Apply Kirchhoff's voltage law

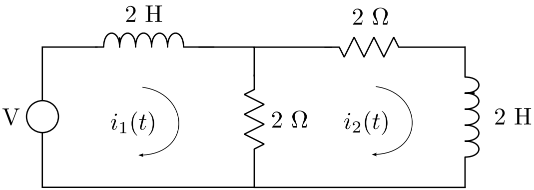
→ Find the equations for i_1 and i_2

Exercise 8 - continued



Exercise 9

Write the the differential equations ($i = f(V)$) of the following circuit.

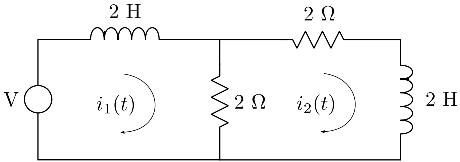


Procedure:

→ Apply Kirchhoff's voltage law

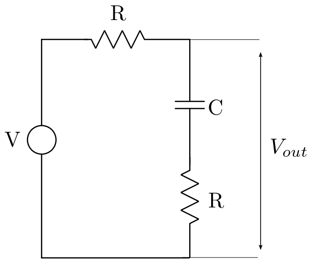
→ Find the equations for i_1 and i_2

Exercise 9 - continued



Exercise 10

Write the the differential equations ($V_{out} = f(V)$) of the following circuit.

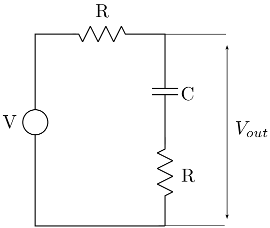


Procedure:

→ Apply Kirchhoff's law

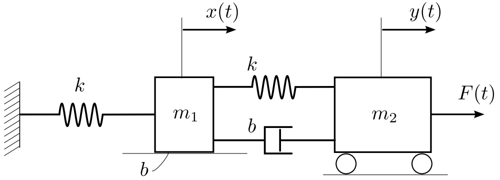
→ Find the equations for V_{out} as a function of V

Exercise 10 - continued



Skills check 1 - Taken from 2018 final examination

A tiny passive micro-electromechanical resonator lies at the core of implantable devices for monitoring and treating aneurisms, a leading cause of heart failure. Pressure changes deflect the transducers diaphragm and shifts the mass resonant frequency, which can be monitored externally. An equivalent model of this sensor is shown below as a mass-spring-damper system. Write the equations of motion of each mass in the Laplace domain. All initial conditions are zero.

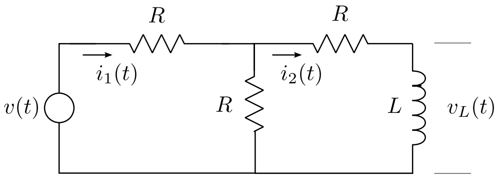


→ $x(t)$ and $y(t)$ are the mass displacements and $F(t)$ is the applied force.[1]

[1] $X(s)(-2k-2bs)+Y(s)(k+bs)=m_1s^2X(s)$, and $F(s)+X(s)(k+bs)+Y(s)(-k-bs)=m_2s^2Y(s)$

Skills check 2 - Taken from 2018 deferred final examination

Find the relations between the input voltage $V(s)$ and the voltage across the inductor $V_L(s)$.



Next class...

- Laplace transform