Impact of Kinematic Structure on the Force Displayability of Planar Passive Haptic Devices

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Abstract—Haptic devices containing passive actuators, such as controllable brakes or dampers, are an attractive alternative to their motor-driven counterparts due to intrinsic stability and improved impedance bandwidth. Passive actuators cannot generate energy and, therefore, the output force can only oppose the applied velocity. In the same way the kinematic structure of traditional manipulators is designed to maximize dexterity and manipulability, one must consider adapting the device topology to optimize force displayability when designing passive actuators.

This paper introduces a set of metrics to evaluate and compare the performance of 2-degree-of-freedom serial and parallel passive haptic devices. These metrics consider the impact of the kinematic structure on the force displayability according to the directions of the device velocity and desired force. Applying these metrics to 9 manipulators revealed that: 1.) Serial manipulators can generate passive forces in all directions equally regardless of the link length ratio; 2.) The base link length of 5-bar parallel manipulators strongly influences passive force displayability; 3.) 5-bar parallel manipulators with the base link length of zero can generate the widest range of passive forces when all links have the same length. The novel performance metrics presented in this paper can aid in the design of 2-DOF passive haptic devices.

Index Terms—Passive Haptics Devices, Passive Actuators, Planar Device, Kinematic Structure, Force Displayability

I. INTRODUCTION

INTERACTIONS with an ideal haptic interface must be indistinguishable from interactions with physical environments. Therefore, stability and transparency are paramount design considerations in any haptic device. Transparency requires a large impedance bandwidth while stability can only be maintained within a limited range [1]-[4]. Haptic devices with passive actuators in place of traditional electric motors are an alternative class of force feedback devices. Passivity of these devices guarantees stability, which in turn, allows them to generate high impedance without requiring high viscous damping. As a result, passive haptic devices can have a wider z-width than their active counterparts. These characteristics make passive haptic devices ideal for applications like robotic teleoperation, rehabilitation, or surgical simulation [5]-[8].

The nonlinearities introduced by the passivity of the brakes make passive devices difficult to control [6], [9]-[11]. Brakes can only generate forces or torques in the direction opposing their velocity. As a result, only the magnitude of the force of a passive device can be controlled, and not the direction [12]. This indicates that a passive haptic device cannot generate a force in an arbitrary direction [13]. The fundamental issue when designing a passive device is maximizing the force output capability, which may depend on its kinematic structure.

There are two main types of manipulator topologies: serial and parallel [3]. Serial manipulators, see Fig. 1(a), are composed of an open chain of actuated links. These manipulators have a larger workspace than their parallel counterparts with similar link lengths. Parallel manipulators, see Fig. 1(b), are formed by connecting two or more serial chains to a common end-effector. Distributing the load onto multiple links minimizes the relative motion of each actuator, resulting in lower apparent inertia of the device [3], [14], increased rigidity, and higher precision than in similar serial manipulators [14].

There are many tools available for examining performance of a manipulator. Condition number, used in [15], and global workspace condition number, proposed in [16], evaluate the accuracy of velocity and force output of a manipulator throughout its workspace. In [17] a unified framework for a holistic analysis of a manipulator, which is simultaneous analysis of acceleration, velocity, and force output capability, was developed using dynamic capability equations. Generalized manipulability ellipsoids, introduced in [18] and adapted for
parallel manipulators in [19], aid in visualizing and quantifying the
ability of a manipulator to generate force in any direction. The
epipods were also used in [20] to determine the optimal pose for a redundant manipulator. The concept of ellipsoids
was adopted for passive devices in [21], where passive manipulability ellipsoids were used to optimize a kinematically
redundant planar 2-degree-of-freedom (DOF) parallel passive
device.

The accuracy and the dynamic performance of serial and parallel manipulators was compared in [14] and [22], respectively. One unexplored issue is the impact of the kinematic structure on the force output capability of a passive device. We address this in this paper by developing a set of metrics measuring the output force capability of a generic passive haptic device. The novel metrics attempt to isolate the effects of actuator passivity by considering the percentage of the
kinematic structure on force displayability. We highlight
serial and parallel manipulators to demonstrate the impact of
directions. The metrics are evaluated on a subset of planar
velocity direction. These metrics complement the metrics for
actuator passivity by considering the percentage of the
structure on the force output capability of a passive device.

We address this in this paper by developing a set of metrics
to a redundant planar 2-degree-of-freedom (DOF) parallel passive
manipulator, see Fig.1(a), the Jacobian is:

$$J = \begin{bmatrix}
- l_2 \sin(\theta_1 + \theta_2) - l_2 \sin(\theta_1 - \theta_2) - l_2 \sin(\theta_1 - \theta_2)

l_2 \cos(\theta_1 + \theta_2) + l_2 \cos(\theta_1 + \theta_2)
\end{bmatrix} \quad (2)$$

[23] and the Jacobian of the parallel manipulator in Fig.1(b) is [24]:

$$J = \begin{bmatrix}
\frac{l_1 \sin(\theta_1 - \theta_3) \sin(\theta_4) - l_2 \sin(\theta_1 - \theta_3) \sin(\theta_4)}{\sin(\theta_4) - \sin(\theta_4)}
\frac{l_1 \cos(\theta_1 - \theta_3) \cos(\theta_4) - l_2 \cos(\theta_1 - \theta_3) \cos(\theta_4)}{\sin(\theta_4) - \sin(\theta_4)}
\end{bmatrix}. \quad (3)$$

The inverse transpose of the Jacobian relates the force in Cartesian space $F \in \mathbb{R}^{n \times 1}$ experienced at the end-effector and joint torques $\tau \in \mathbb{R}^{n \times 1}$ such that

$$F = (J^{-1})^T \tau \quad (4)$$

where each column of the inverse transpose Jacobian relates to a reference force at the end-effector i.e.,

$$\begin{bmatrix}
R_{x_1} & R_{x_2} & \cdots & R_{x_n}
\end{bmatrix}
\begin{bmatrix}
R_{y_1} & R_{y_2} & \cdots & R_{y_n}
\end{bmatrix}
\begin{bmatrix}
R_{z_1} & R_{z_2} & \cdots & R_{z_n}
\end{bmatrix}
= \begin{bmatrix}
R_1 & R_2 & \cdots & R_n
\end{bmatrix} \quad (5)$$

Column 1 represents force vector $R_1$ as shown in Fig.1. Each column corresponds to a reference force $R_i$, resulting from applying unit torque $\tau_j$ to a single joint $j$ at a time. Summing the reference forces provides an overall output force $F_{a}$ at the end-effector for a generic manipulator i.e.,

$$F_{a} = \sum_{j=1}^{n} a_j R_j \quad (6)$$

where $a_j \in \mathbb{R}$ represents scaling factors on each reference force. These scaling factors provide a convenient method of visualizing the forces that a passive device generates and they aid in approximating forces. Note that (6) and (4) are equivalent, thus, the torque $\tau_j = a_j$.

Consider Fig.1(a) and (b), respectively demonstrating a dual-actuated serial manipulator and a five bar parallel ma-

nicipulator, each with two controllable DOF. A combination of the reference forces $-R_1$ and $-R_2$ corresponds to the force resulting from negative torques $\tau_1 = -1, \tau_2 = 0$, and $\tau_1 = 0, \tau_2 = -1$, respectively. To generate force $F_2$, shown in
Fig.1(c), a combination of both $-R_1$ and $-R_2$ must be used.

In an ideal device, $V \in \mathbb{R}^{n \times 1}$ $\exists \left [ a_1, a_2, \cdots a_n \right ] \in \mathbb{R}$ such that $F_a = F_d$. In non-ideal devices, however, saturation exists in each actuator and therefore $a_j$ is bound to the maximum achievable torque $\tau_{max,j}$ of an actuator $j$. Introduc-
saturating each actuator generates a range of possible output forces $\Omega_1$ at the end-effector, as outlined in Fig.1(d) where contour $\Omega_1$ represents the maximum achievable force in any direction. This region enclosed by $\Omega_1$ represents an arbitrary aggregation of the reference forces, which includes all forces the device can generate.

In the context of passive devices, the only forces that can be created must act against the angular velocity of the brake and in the direction of the reference force $R_j$ i.e., $\theta_j \tau_j < 0$. For devices with multiple passive actuators, the collaboration

$$V = J \dot{\theta} \quad (1)$$

where $1 \leq j, k \leq n$. For a non-redundant manipulator, the Jacobian results in an $n \times n$ matrix. The forward kinematics determine the position of the end-effector for a given set of
joint angles, while the inverse kinematics are used to find joint
angles for a given end-effector position. For the 2-DOF serial
manipulator, see Fig.1(a), the Jacobian is:

$$J = \begin{bmatrix}
- l_2 \sin(\theta_1 + \theta_2) - l_2 \sin(\theta_1 - \theta_2)

l_2 \cos(\theta_1 + \theta_2) + l_2 \cos(\theta_1 + \theta_2)
\end{bmatrix} \quad (2)$$

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Venom encompasses Ω this method. This region is a subset of Ω as the Heaviside function. The i.e., sgn( proximation is only achievable if the projection of the desired force onto a displayable reference force, like \( F \), determines the regions in which components of an arbitrary partial force display of the force output. One of these methods, passive haptic device can create while sacrificing the precision of the device. To evaluate the performance of passive devices, it is necessary to expand the analysis to the workspace of the manipulators.

III. PERFORMANCE METRICS FOR PASSIVE HAPTIC DEVICES

To better understand the impact of the kinematic structure on force output capability of a passive haptic device one must isolate the effects of actuator passivity from the dynamics of the device. To this end, let us consider the area of the workspace where a given force can be either fully or partially-displayed as a performance metric.

The total workspace of a device \( A_0 \) can be divided into 3 regions: \( A_0, A_p, \) and \( A_n \), where subscript \( d, p \) and \( n \) correspond to fully, partially, and non-displayable regions, respectively, see Fig. 2. The displayability of the force at each point depends on directions of velocity and desired force. As shown in Fig. 3(a), the directions of the velocity are defined as an angle \( \alpha \) which for planar devices has one component, the angle from positive x axis, and for spatial devices two components, along x and y axes. The direction of the desired force is defined as an angle \( \phi \) separating velocity and the desired force along the same axes as \( \alpha \). Ideally, the device must be able to generate a force in any direction, for any velocity, everywhere in its workspace. A passive device, however, can only generate the desired force in some parts of its workspace.

The total workspace area of a device can be found by integrating all of the points in the workspace. Since the geometry of the workspace resembles a circle, the total workspace area is found with a polar integral such that,

\[
A = \int_{0}^{\alpha} \int_{0}^{\beta} r \, dr \, d\sigma
\]

where \( r = ||P(r, \theta, \sigma)|| \) is a point within the device’s workspace, \( \theta, \sigma \), and \( r \) are the polar coordinates. The area of the region where a force can be fully displayed or approximated is found by integrating only the points satisfying the conditions given by (8). Currently, there is no analytical formulation to describe fully or partially displayable points. As a result, the area of these regions is approximated by,

\[
A \approx \sum_{j=1}^{n} \sum_{k=1}^{m} \sum_{i=1}^{p} ||P(r_i, \theta_k, \sigma_j)|| \, \Delta r \, \Delta \theta \, \Delta \sigma
\]

where \( P(r_i, \theta_k, \sigma_j) \) represents a point in the workspace satisfying the desired constraint.

To compare multiple types of manipulators with differing workspaces the area of each region must be normalized

\[
\mu = \frac{A}{A_0}
\]
Let the mean value of the components angle between velocity and the desired force along one of the manipulator can display a desired force in any direction. In this paper we analyze only planar robots, thus, of each topology, thus, aiding in the design of a passive device.

The performance of serial and parallel manipulators are evaluated using metrics in (11), (12), and (13). To reduce the number of possible kinematic solutions and singularities in the workspace of the two manipulators, both manipulators were constrained to half of the reachable workspace. The analysis also assumed all parallel manipulators to be symmetric i.e., \( l_1 = l_2 \) and \( l_3 = l_4 \), with \( l_5 = 0 \) or 1. Link lengths of the two manipulators are normalized and only the link length ratio, \( R = l_1/l_3 = l_2/l_4 \) for parallel and \( R = l_1/l_2 \) for serial manipulators, varies. There are 3 general cases to consider for each manipulator \( R = 0.5, 1, 2 \). In total, 9 manipulator configurations, listed in Table I were considered.

Their performance is aggregated and summarized in Table I, while the detailed results are divided into three sets. Fig.3(b) shows the performance measures for all serial manipulators. The results for parallel manipulators are grouped by length of \( l_5 \). Fig.4(a) and (b) shows the performance results of parallel manipulators with \( l_5 = 0 \). For manipulators with \( l_5 = 1 \), on the other hand, the performance is shown in Fig.4(c) and (d) as well as, Fig.5 and Fig.6.

V. Discussion

There are clear distinctions in performance across the spectrum of manipulators. All serial manipulators performed the same, irrespective of their link length ratio \( R \) or direction of velocity \( \alpha \). The performance of parallel manipulators varied significantly as a function of both \( R \) and \( l_5 \). As a result, the performance of parallel manipulators will be analyzed separately for manipulators with \( l_5 = 0 \) and \( l_5 = 1 \).

A. Serial Manipulators

The results for all serial manipulators, independent of the link length ratio or the direction of velocity were the same as shown in Table I. Let \( \mu_{d_{-max}} = \max \{ \mu_d \} \forall \alpha \), \( \forall \phi \), represent the highest percentage of the fully displayable region \( \mu_d \). For a serial manipulator, this angle occurs when the angle between the velocity and the desired force is \( \phi = \pi \), as shown in Fig.3(b). Note that since \( \mu_{d_{-max}} < 1 \), there are parts of the workspace where a force directly opposing the velocity cannot be rendered. When \( \phi = \pi \), \( \mu_d = 0.63 \) and \( \mu_p = 0.37 \), which means that desired force can be generated in 63% of the device’s workspace while in the remaining 37% the force can be approximated. Since \( \tilde{\eta}_d = \mu_d = 0.179 \) and \( \tilde{\eta}_p = \eta_p = 0.5 \), the performance of these manipulators does not vary with \( \alpha \). Consequently, for all combinations of \( \alpha \), the mean percentage of the fully and partially displayable regions in the workspace was 17.9% and 50.0% respectively. All serial manipulators, therefore, can fully or partially display 70.3% of all forces irrespective of the velocity direction.
B. Parallel Manipulators with $l_5 = 0$

Like serial manipulators, the performance of parallel manipulators with base link length $l_5 = 0$ is insensitive to change in $\alpha$, meaning that $\eta_d = \eta_d$ and $\eta_p = \eta_p$. Unlike serial manipulators, the performance varies depending on the link length ratio $R$, which is optimal when $R = 1$. Their optimal values are exactly the same as all serial manipulators, as shown in Fig. 4(a) and (b).

Any link ratio other than $R = 1$ deteriorates $\mu_d$, $\mu_p$, $\eta_d$, and $\eta_p$. As shown in Table I and Fig. 4(a) and (b), the manipulator with $R = 2$ has the lowest maximum $\mu_d$ and $\mu_p$, and the lowest $\eta_d$ and $\eta_p$, meaning it could generate a smaller range of forces than the manipulators with $R = 1$ or 0.5. From Fig. 4(a) and (b), the manipulator with $R = 1$ performed slightly better than manipulator with $R = 0.5$. This change is reflected in the maximum value of $\mu_d$ listed in Table I. However, the difference in $\eta$ and $\bar{\eta}$ of the two manipulators is insignificant given the working precision. Thus, the link lengths of this manipulator must be equal to maximize performance, and there is no advantage to varying the link length from the perspective of a passive haptic device.

C. Parallel Manipulators with $l_5 = 1$

The performance of parallel manipulators with $l_5 = 1$ varies as a function of $\alpha$ and $\phi$. To start, consider $\mu_d$ and $\mu_p$ of the device for $\alpha = \pi/2$, shown in Fig. 4(c) and (d), respectively. This manipulator is much more sensitive to changes in link ratio $R$. The maximum fraction of the partially displayable region $\mu_d$ for the 3 manipulators varies from 0.639 to 0.97. For some velocity directions, these manipulators can create a force that opposes velocity anywhere in the workspace. The results change significantly depending on the angle $\alpha$. From Fig. 6(a), the maximum value of $\mu_d$ for all angles of $\phi$ is shown as a function of $\alpha$. Notice that at both $\alpha = 0$ and $\alpha = \pi/2$, the maximum $\mu_d$ are the same, for each of the three manipulators. This shows that at $\alpha = 0, \pi/2, \pi,$ and $3\pi/2$, the manipulator with $R = 2$ can generate a force at any point

in its workspace. The peak value of $\mu_d$ for this manipulator, however, experiences the steepest drops, as shown in Fig. 6(a).

Let us focus on Fig. 6(b), which shows the variation of average fraction of the displayable region $\mu_d$ is $\eta_d$, for the three manipulators as a function of $\alpha$. Notice that, compared with Fig. 6(a), peaks of $\mu_d$ and $\eta_d$ do not align. To investigate the discrepancy, let us examine Fig. 5, where $\mu_d$ and $\mu_p$ are shown for the three manipulators, at $\alpha = 0, \pi/4,$ and $\pi/2$. Fig. 5(a) (c) and (e), all show that for $\alpha = 0$ and $\alpha = \pi/2$, the performance has the same peak $\mu_d$. As the curves of $\mu_d$ and $\mu_p$ illustrate at $\alpha = \pi/2$ all 3 of the manipulators can render more forces than at $\alpha = 0$ in directions other than $\phi = \pi/2$. The peaks in Fig. 6(a) that do not correspond with peaks in (b), represent directions where force at a specific direction, can be generated almost anywhere in the workspace, but all other forces will be difficult to render.

Lastly, consider the variation in performance of the 3 manipulators, shown in Fig. 6. The performance, $\mu_d_{-\text{max}}$ and
\[ \eta_d \] clearly varies the least in the manipulator with \( R = 1 \). The manipulator with \( R = 2 \), on the other hand, has the highest peak performances, but also steepest declines. The performance of the manipulator with \( R = 0.5 \) is almost as high as for the manipulator with \( R = 2 \), but it does not have as steep of declines i.e., \( \frac{\partial \eta_d}{\partial \alpha} \). As a result, this manipulator performs the best, as indicated by a high value of \( \eta_d \) listed in Table I. Notice that all parallel manipulators with both manipulators with \( R = 1 \) and 0.5, had the highest value of \( \eta_d \) of all manipulators.

VI. CONCLUSIONS

This paper presents the first analysis of the effects of the kinematic structure on the ability of a passive haptic device to generate forces. The performance metrics introduced here evaluate the areas of the device’s workspace where the desired force can be either fully displayed, or partially displayed.

The analysis considers 9 manipulators, 3 serial and 6 parallel kinematic chains, with link length ratios of 0.5, 1, and 2. The following recommendations can be made for 2-DOF RR serial and symmetric 2-DOF five-bar parallel manipulators from the findings of this paper:

- Passive serial manipulators generate forces in all directions equally well independent of the link length ratio;
- Parallel manipulators with a base link length of zero generate the widest range of forces in their workspace when all links have the same length;
- Parallel manipulators with a base link length equal to one perform better in a larger percentile of the workspace than serial manipulators;

This performance, however, varies significantly depending on the direction of the velocity. Parallel manipulators perform best when the velocity acts perpendicular to the base of the manipulator, and worst when the velocity acts at an angle of \( \pi/4 \) from the base link. In this category of manipulators, the one with link length ratio of 0.5, performs the best.

Serial manipulators perform equally well for all combinations of link length ratios, so no optimal configuration of this manipulator exists in terms of force displayability. For parallel manipulators there exists no global optimum solution; force displayability can only be improved for certain velocity directions. Thus, optimization of a parallel manipulator should be conducted on a case-by-case basis. During the design, the metrics presented here should be used in conjunction with other dexterity metrics to develop a fitness equation tailored to the application. These results aid in the design of 2-DOF passive haptic devices. Serial and parallel manipulators with \( l_3 = 0 \) are equally suitable for applications where the user is expected to move in all directions equally i.e., the most general use case. For specialized applications where the motion has a predominant direction, parallel manipulators with \( l_3 > 0 \) are a better choice. These applications include simulation of needle insertion tasks [25], teleoperation of robots in constrained workspaces, or upper limb patient rehabilitation [26].

The metrics proposed in this paper should be used in conjunction with other metrics, like manipulability [18], [19], dynamic capability equations [17] and workspace condition number [15], [16], to design a passive haptic device with the desired force output capability and dynamic characteristics.

REFERENCES