MECE 2230U
Statics

Lecture 18
Dry Friction - 3/3

Outline of Lecture 18

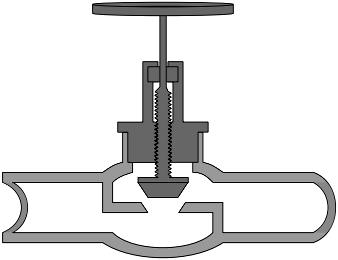
By the end of today's lecture you should be able to

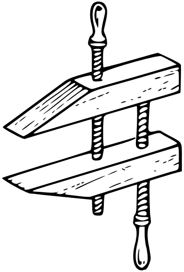
- Determine the forces on screws and bearings
- Determine the moments on screw and bearings
- Solve equilibrium problems involving friction

Applications

Screws can be used as fastener but also to transmit power or motion.

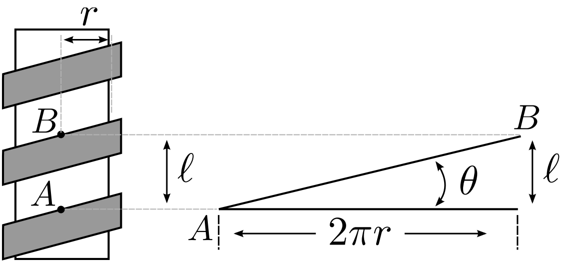
How do we find the moment required to turn the screw ?





Frictional forces on screws

The square-threaded screw can be considered as a cylinder having and included square thread wrapped around it.



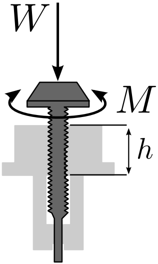
where:

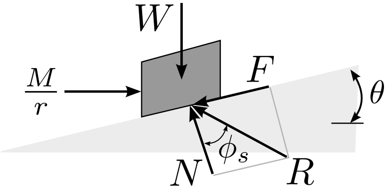
→ $\theta = \tan^{-1}(\frac{\ell}{2\pi r})$ is the slope or lead angle

→ ℓ is the lead of screw

→ r is the mean radius

Upward impeding motion





where:

→ M couple moment about the shaft

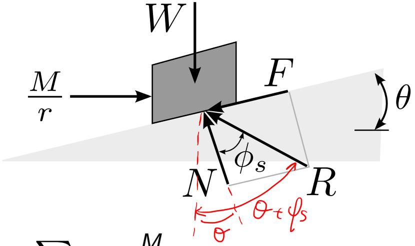
→ $P = \frac{M}{r}$ vertical force acting on the thread

→ R resultant reaction: normal reaction N and friction $F = \mu_s N$

→ $\phi_s = \tan^{-1}(\frac{F}{N}) = \tan^{-1}(\mu_s)$ angle of static friction

Upward impeding motion

Equations of equilibrium



$$\sum F_x = \frac{M}{r} - R\sin(\theta + \phi_s) = 0 \qquad (1)$$

$$\sum F_y = R\cos(\theta + \phi_s) - W = 0 \qquad (2)$$

Combining (1) and (2) gives:

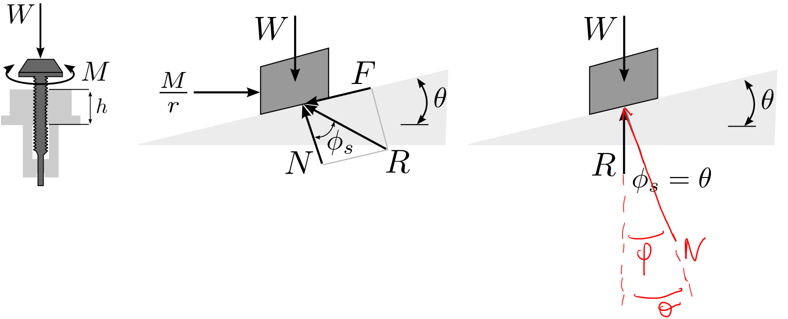
$$M = rW\tan(\theta + \phi_s) \qquad (3)$$

Self-locking screw

→ A screw is self-locking if it remains in place for $\mathbf{M} = 0$.

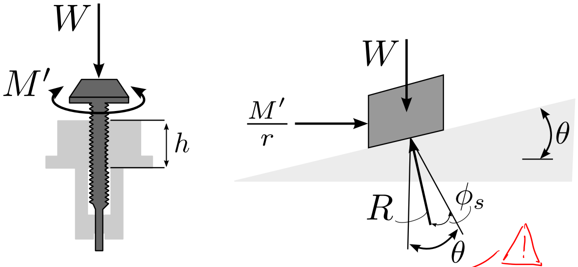
→ If $M = 0$, the direction of **F** is reversed and acts on the other side of **N**.

→ If $\phi_s = \theta$ then **R** balances **W**.



Downward impeding motion

Case 1 - The screw is not self-locking: A moment M' is required to prevent downward winding.



$$\sum F_x = \frac{M'}{r} - R\sin(\theta - \phi_s) = 0 \tag{4}$$

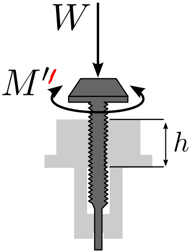
$$\sum F_y = R\cos(\theta - \phi_s) - W = 0 \tag{5}$$

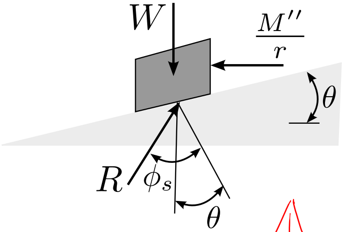
Combining (4) and (5) gives:

$$M' = rW\tan(\theta - \phi_s) \tag{6}$$

Downward impeding motion

Case 2 - The screw is self-locking: A moment M'' is required to wind it downward.





$$\sum F_x = \frac{M''}{r} - R\sin(\phi_s - \theta) = 0 \quad (7)$$

$$\sum F_y = R\cos(\phi_s - \theta) - W = 0 \quad (8)$$

Combining (4) and (5) gives:

$$M'' = rW\tan(\phi_s - \theta) \quad (9)$$

Exercise 1

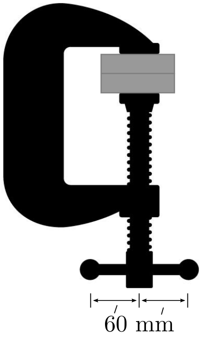
Find the magnitude of the couple forces that must be applied to the lever of the clamp in order to loose the screw. The screw has a diameter of 10 mm, a lead of 2.5 mm and $\mu_s = 0.3$. The clamping force is 600 N.

Procedure

→ Calculate the angle of static friction and the slope

→ Determine the required moment

→ Determine the required force



Exercise 1 - continued

Screw slope:

$$\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) \rightarrow \tan^{-1}\left(\frac{2.5\ mm}{2\pi\ 5\ mm}\right) = 4.55^\circ$$

Angle of static friction:

$$\phi_s = \tan^{-1}(\mu_s) = \tan^{-1}(0.3) = 16.7^\circ$$

case 2

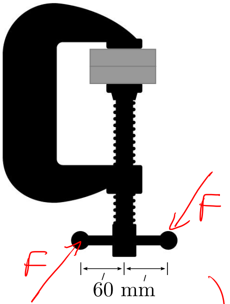
Required moment:

$$M = r\, W \tan(\phi_s - \theta)$$

tan, not $\tan^{-1}$

$$M = 5\ mm\ (600\ N) \tan(16.7^\circ - 4.55^\circ)$$

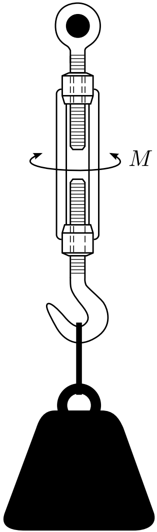
$$M = 645.8\ Nmm$$



$$F = \frac{M}{120\ mm} \rightarrow F = 5.38\ N$$

Exercise 2

The turnbuckle has a square thread with a mean radius of 5 mm, a lead of 2 mm and $\mu_s = 0.25$. Determine the moment that must be applied to lift a 2kN weight.



Procedure

→ Calculate the angle of static friction and the slope

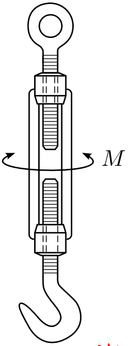
→ Determine the required moment

Exercise 2 - continued

$$M = 2[r\,W \tan(\theta + \phi_s)]$$
↳ two screws.

Screw slope:

$$\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left(\frac{2}{2\pi 5}\right) = 3.64^\circ$$



Angle of static friction:

$$\phi_s = \tan^{-1}(0.25) = 14.04^\circ$$

Required moment:

$$M = 2(2000\,N \cdot 5\,mm)\tan(14.04^\circ + 3.64^\circ)$$

$$\boxed{M = 6.37\,Nm}$$ or $6374.7\ N\,mm$

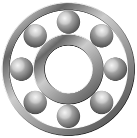
What happens when the moment is removed ?

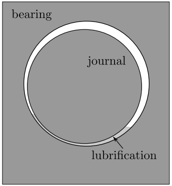
Since $\phi_s > \theta$, the screw will not unscrew → self locking.

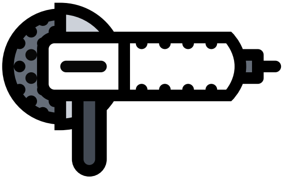
Frictional forces on bearings

Bearings are used to support axial load on a rotating shaft.

If bearings are not lubricated, the laws of friction are applied to find the moment required to turn the shaft.







Collar bearing

→ Contact area:

$$A = \pi(R_2^2 - R_1^2)$$

→ Normal pressure:

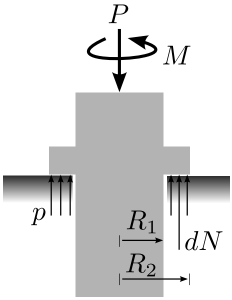
$$p = \frac{\mathbf{P}}{A} = \frac{\mathbf{P}}{\pi(R_2^2 - R_1^2)}$$

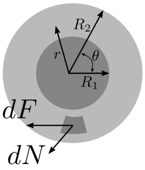
→ For a differential area element:

$$dA = r(d\theta)(dr)$$
$$dN = p(dA)$$

→ The friction force is:

$$dF = \mu_s dN = \mu_s p dA$$
$$dF = \frac{\mu_s}{\pi}\frac{\mathbf{P}}{R_2^2 - R_1^2} dA$$





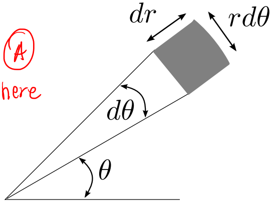
Collar bearing

→ The moment about the z axis is:

$$dM = r(dF)$$

$$M - \int_A r\,dF = 0$$

replace (A) here



→ Integrating over the surface gives:

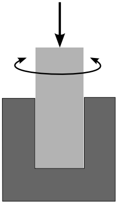
$$M = \int_{R_1}^{R_2}\int_0^{2\pi} r\left[\frac{\mu_s P}{\pi(R_2^2 - R_1^2)}\right] r\,d\theta\,dr = \frac{\mu_s P}{\pi(R_2^2 - R_1^2)}\int_{R_1}^{R_2} r^2\,dr \int_0^{2\pi} d\theta$$

$$M = \frac{2}{3}\mu_s P\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

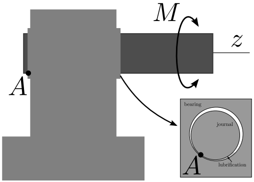
→ For a pivot bearing:

$$R_2 = R$$
$$R_1 = 0$$

thus $M = \frac{2}{3}\mu_s P R$



Journal bearing



→ The reaction at A is opposed to the applied force **P**.

→ The net moment about z is zero $\sum M_z = 0$.

$$\mathbf{M} - \mathbf{r}_f \times \mathbf{R} = 0$$

$$M - R(r \sin \phi_k) = 0$$

→ $\tan(\phi_k) = \frac{F}{N} = \frac{\mu_k N}{N} = \mu_k$ is the angle of kinetic friction.

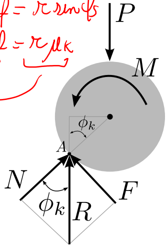
→ If $\mu_k \to 0$, then $\sin \phi_k \approx \tan \phi_k \approx \mu_k$ and → $r_f = r \sin \phi_k$

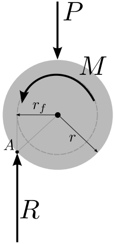
$r_f = r \mu_k$

$$M \approx Rr\mu_k$$

→ To minimize friction, r should be small.

→ To minimize friction, μ_k should be small.





Exercise 3

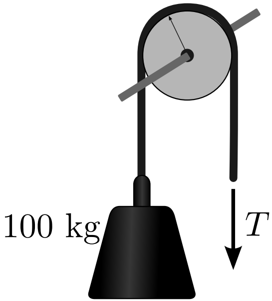
The 100 mm diameter pulley fits on a 10 mm diameter shaft for which $\mu_s = 0.4$. Determine the tension T needed to raise the 100 kg weight.

Procedure

→ Determine the friction circle

→ Draw the free-body diagram

→ Apply the equations of equilibrium

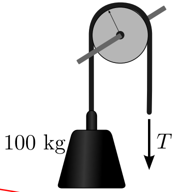


Exercise 3 - continued

→ Friction circle

$$r_f = r\sin(\varphi_s) \cong r\tan(\varphi_s) \simeq r\mu_k$$

$$r_f = 5\,(0.4) = \boxed{2\,mm}$$

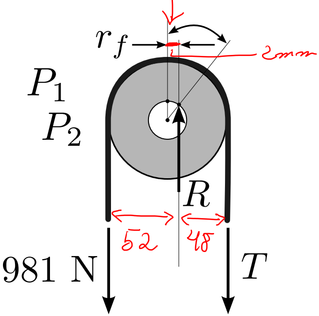


→ Movements about P_2

$$\Sigma M_{P_2} = 0$$

$$981\,(52) - T\,(48) = 0$$

$$\boxed{T = 1063\ N}$$



Next class...

- Centre of gravity
- Please take a look at lecture 19