METE 3100U
Actuators and Power Electronics

Lecture 4
Diode Rectifiers

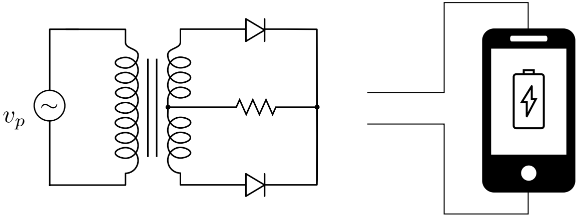
Outline of Lecture 4

In today's lecture we will

- Review the principles of diodes
- Analyse and design diode rectifier circuits
- Determine the effects of load inductance on the load current

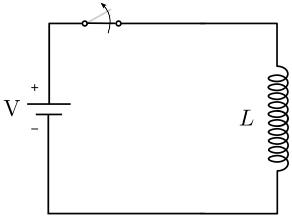
Applications

A rectifier converts alternating current to direct current. The process is known as rectification, since it "straightens" the direction of current.



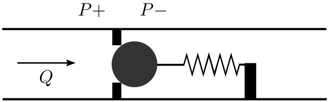
Applications

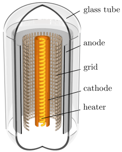
For highly inductive load, who can the energy stored in the inductor be dissipated when the circuit is turned off?

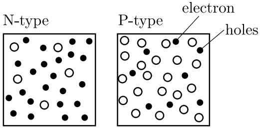


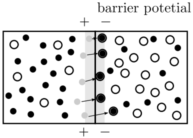
Diode working principle

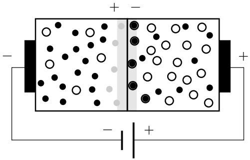
A diode is a semiconductor with two terminals allowing the flow of current in one direction only.

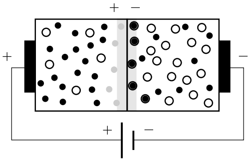




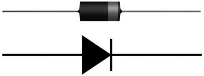








Ideal *vs* practical diodes

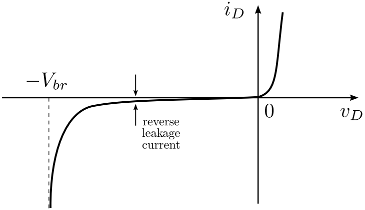


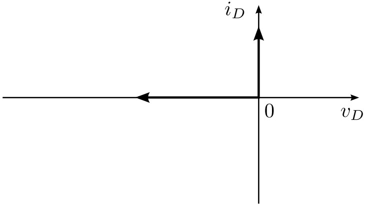
Operation modes:

→ Forward-biased region: $v_D > 0$, the diode conducts

→ Reverse-biased region: $v_D < 0$, the diode does not conduct

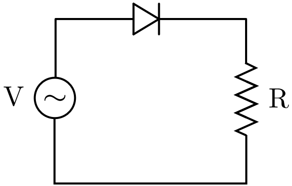
→ Breakdown region: $v_D < -V_{br}$, the current is reversed

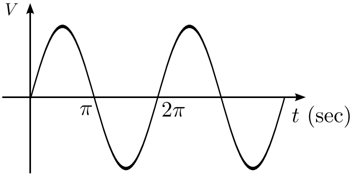


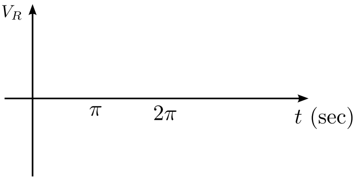


Diode rectification

Diodes can be used to convert an alternating voltage (AC) into a continuous voltage (DC)



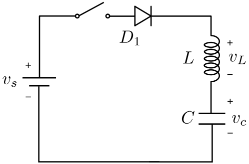




The average output voltage is

$$V_R = \frac{1}{T}\int_0^T V(t)dt \tag{1}$$

Diode-switched LC load



$$v_s = L\frac{di}{dt} + \frac{1}{C}\int_{t_0}^{t} idt + v_c(t=0) \tag{2}$$

With no initial conditions ($i(0) = 0$, $v_c(0) = 0$) the Laplace transform gives:

$$V_s(s) = LI(s)s + \frac{I(s)}{Cs} \rightarrow I(s) = \frac{V_s}{L(s^2 + \omega_n^2)} \tag{3}$$

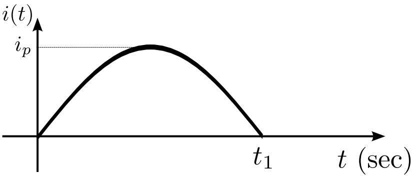
where $\omega_n = 1/\sqrt{LC}$ and $i(t) = \mathcal{L}^{-1}I(s)$, thus:

$$i(t) = v_s\sqrt{\frac{C}{L}}\sin(\omega_n t) \tag{4}$$

Diode-switched LC load

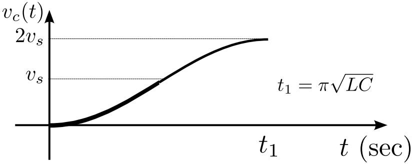
The current is:

$$i(t) = v_s \sqrt{\frac{C}{L}} \sin(\omega_n t) \tag{5}$$



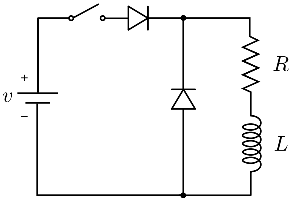
And the voltage v_c across the capacitor is:

$$v_c(t) = \frac{1}{C} \int_0^t i dt = v_s(1 - \cos(\omega_n t)) \tag{6}$$

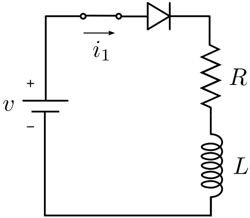


Freewheeling diodes

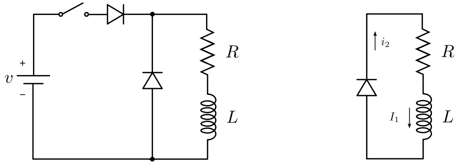
$\rightarrow$ Eliminates the sudden voltage spike across inductive loads when its supply current is suddenly reduced.



Circuit **mode 1** - The switch is closed. The current is:



Freewheeling diodes

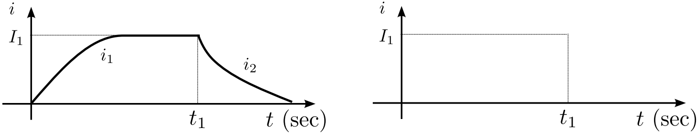


Circuit **mode 2** - The switch is opened and $i_L = I_1$. The current is:

$$0 = L\frac{di_2}{dt} + Ri_2 \tag{7}$$

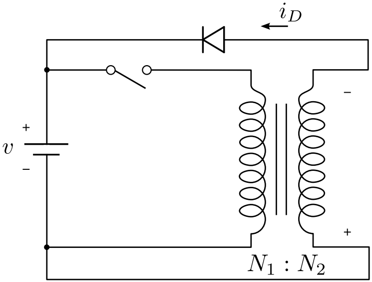
With the initial condition $i_2(0) = I_1$, the current becomes

$$i_2(t) = I_1 e^{-\frac{R}{L}t} \tag{8}$$



Recovery of trapped energy

The trapped energy in the transformer can be returned to the primary source.

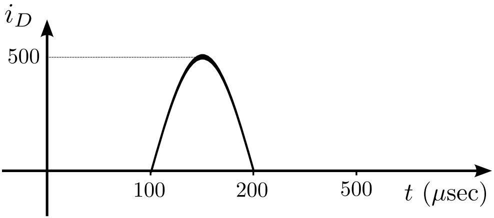


Mode 1 - The switch is closed: $i_D = 0$.

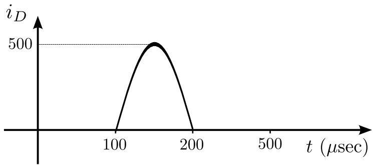
Mode 2 - The switch is open: The voltage across the inductor is reversed: $i_D \neq 0$

Exercise 10

The current waveform of a diode is shown in the figure. Determine the average, and the peak current of the diode. Assume a half sine wave.

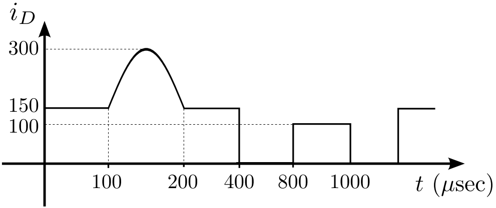


Exercise 10 - continued

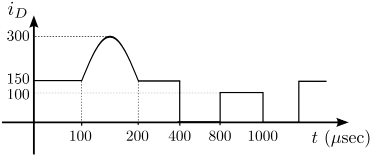


Exercise 11

The waveforms of the current flowing through a diode is shown in the figure. Determine the average, rms, and peak current ratings of the diode. Assume a half sine wave of 150 A (peak)

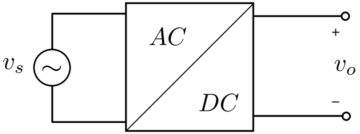


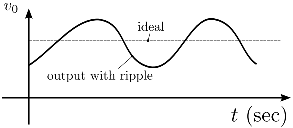
Exercise 11 - continued



Diode rectifiers

Rectifiers convert AC current to DC current





→ A rectifier is a power processor

→ Input voltage and current must remain in phase

Output AC power:

$$P_{ac} = V_{mrs} I_{rms} \tag{9}$$

Output DC power:

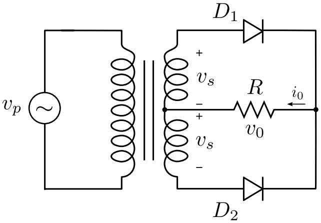
$$P_{dc} = V_{dc} I_{dc} \tag{10}$$

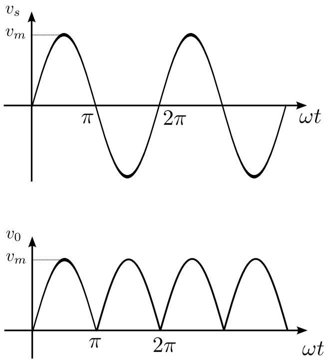
Rectification ratio:

$$\eta = \frac{P_{dc}}{P_{ac}} \tag{11}$$

Single phase full wave rectifiers

Centre-trapper transformer rectifier: An additional wire is connected across the middle of the secondary winding of a transformer





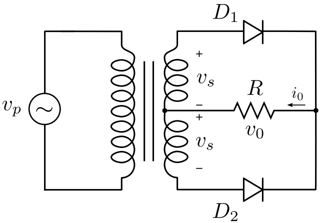
The average load voltage is

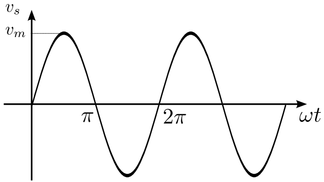
$$V_0 = \frac{2}{T}\int_0^{\frac{T}{2}} v_s \sin(\omega t) d\omega t$$

$$V_0 =$$

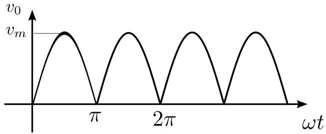
Single phase full wave rectifiers

Centre-trapper transformer rectifier: An additional wire is connected across the middle of the secondary winding of a transformer





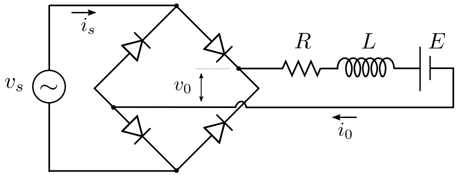
The root mean square (rms) voltage is



$$V_0 = \sqrt{\frac{2}{T}\int_0^{\frac{T}{2}} [v_s \sin(\omega t)]^2 d\omega t}$$

$$V_0 = \frac{V_m}{\sqrt{2}} = 0.707 v_s$$

Bridge rectifier with RL load

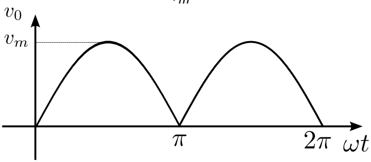


The generator voltage is

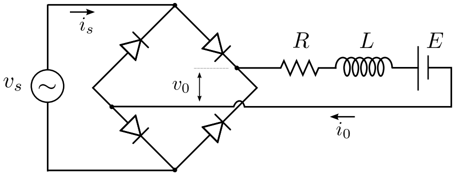
$$v_s = (V_s\sqrt{2})\sin\omega t \tag{12}$$

where V_s is the RMS input voltage. The voltage v_0 across the load

$$v_0 = |\underbrace{(V_s\sqrt{2})}_{v_m}\sin\omega t| \tag{13}$$



Bridge rectifier with RL load



The load current is

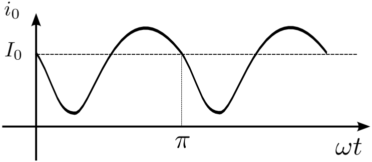
$$L\frac{di_0}{dt} + Ri_0 + E = |\sqrt{2}V_s \sin\omega t| \ \forall\ i_0 > 0 \tag{14}$$

which has a solution of the form

$$i_0 = \left| \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \theta) + ke^{-\frac{R}{L}t} - \frac{E}{R} \right| \tag{15}$$

with $Z = \sqrt{R^2 + (\omega L)^2}$ and $\theta = \tan^{-1}(\omega L/R)$.

Bridge rectifier with RL load



I_0 is the steady state current. At $\omega t = \pi$, we set $i_0 = I_0$, thus

$$k = \left(I_0 + \frac{E}{R} - \frac{\sqrt{2}V_s}{Z} \sin\theta \right) e^{\frac{R}{L}\frac{\pi}{\omega}} \tag{16}$$

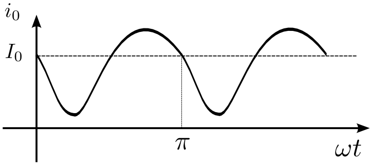
Replacing the above in (15) yields:

$$i_0 = \frac{\sqrt{2}V_s}{Z} \sin(\omega t - \theta) + \left(I_0 + \frac{E}{R} - \frac{\sqrt{2}V_s}{Z} \sin\theta \right) e^{\frac{R}{L}\left(\frac{\pi}{\omega} - t\right)} - \frac{E}{R} \tag{17}$$

Steady-state: $I_0 = i_0(\omega t = 0) = i_0(\omega t = \pi)$

$$I_0 = \frac{\sqrt{2}V_s}{Z} \sin(\theta) \frac{1 + e^{-(R/L)(\pi/\omega)}}{1 - e^{-(R/L)(\pi/\omega)}} - \frac{E}{R} \tag{18}$$

Bridge rectifier with RL load



Finally, the current across the load is

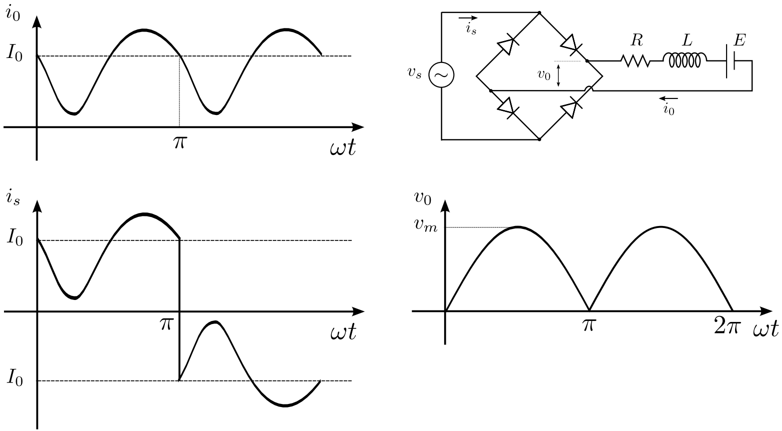
$$i_0 = \frac{\sqrt{2}V_s}{Z}\left[\sin(\omega t - \theta) + \frac{2}{1 - e^{-(R/L)(\pi/\omega)}}\sin\theta\left(e^{-\frac{R}{L}t}\right)\right] - \frac{E}{R} \qquad (19)$$

with $Z = \sqrt{R^2 + (\omega L)^2}$ and $\theta = \tan^{-1}(\omega L/R)$.

The average current i_{av} and rms current i_{rms} are

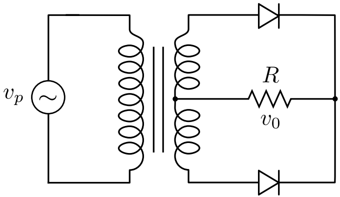
$$i_{av} = \frac{1}{2\pi}\int_0^{\pi} i_0 d(\omega t) \qquad i_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i_0^2 d(\omega t)} \qquad (20)$$

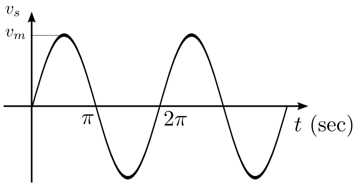
Bridge rectifier with RL load



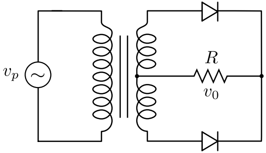
Exercise 12

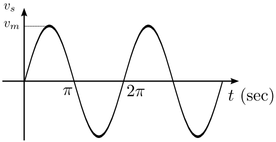
The single phase bridge rectifier has a purely resistive load $R = 5\Omega$, a peak supply voltage $V_m = 170$ V, and a supply frequency of 60 Hz. Determine the average output voltage of the rectifier if the source inductance is negligible.





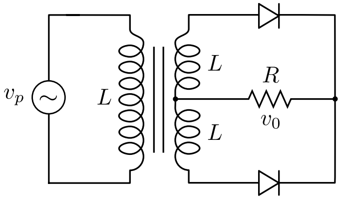
Exercise 12 - continued

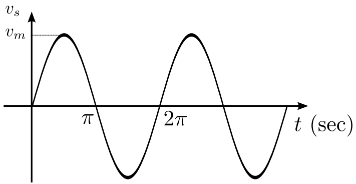




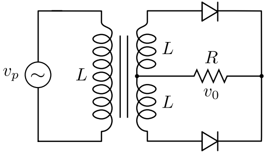
Exercise 13

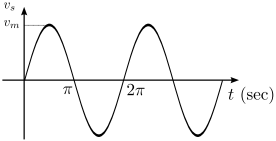
The single phase bridge rectifier has a purely resistive load $R = 5\Omega$, a source inductance per phase including the transformer leakage inductance of 0.5 mH, a peak supply voltage $V_m = 170$ V, and a supply frequency of 60 Hz. Determine the average output voltage of the inductor.





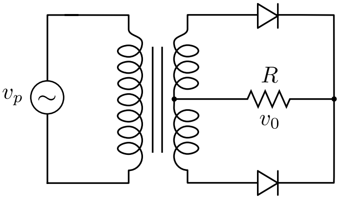
Exercise 13 - continued

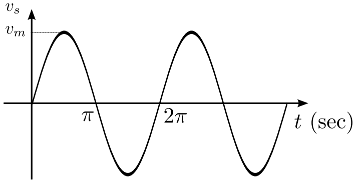




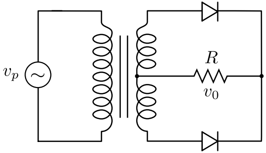
Exercise 14

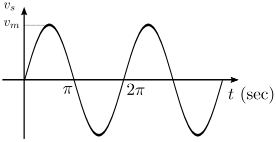
The bridge rectifier shown is required to supply an average voltage of $V_{dc} = 240$ V to a resistive load of $R = 10\Omega$. Determine the required kVA rating of the transformer. Neglect all inductances.



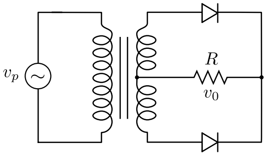


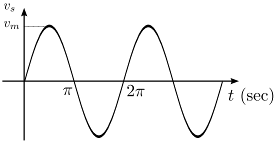
Exercise 14 - continued





Exercise 14 - continued





Next class...

- DC/DC converters

Additional supporting materials for Lecture 4:

Diode working principle: `https://goo.gl/mncLJR`

RMS and average voltages: `ttps://goo.gl/xaiN2C`