MECE 3350U
Control Systems

# Lecture 6
# Block Diagram Models

## Videos in this lecture

Lecture: https://youtu.be/Eq2eGyHNlxA

Exercise 23: https://youtu.be/vInyAfI__xk

Exercise 24: https://youtu.be/xDgSwoGRjJ0

Exercise 25: https://youtu.be/NEmX8l3KNQ0

Exercise 26: https://youtu.be/jJbyqx17VdA

Exercise 27: https://youtu.be/ozfYKqDvrz0

Exercise 28: https://youtu.be/brINH6I5FJg

Exercise 29: https://youtu.be/EEV7lEzCiG0

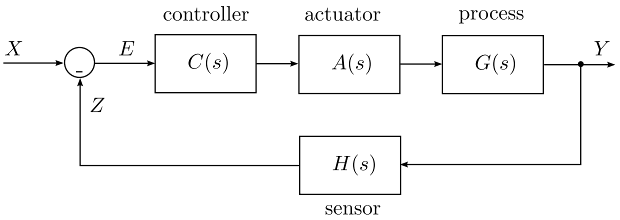
# Outline of Lecture 6

By the end of today's lecture you should be able to

- Represent a control system using block diagrams
- Simply block diagrams
- Find the open-loop transfer function of a closed-loop system

# Applications

What the transfer function of the closed-loop system shown ?



# Applications

The position control system for a spacecraft platform is governed by the following equations:

$$\frac{d^2p(t)}{dt^2} + 2\frac{dp(t)}{dt} + 4p(t) = \theta(t)$$
$$v_1(t) = r(t) - p(t)$$
$$\frac{d\theta(t)}{dt} = 0.5v_2(t)$$
$$v_2(t) = 8v_1(t)$$

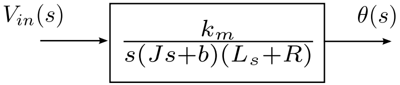
$r(t)$: desired position
$p(t)$: current position
$v_1(t)$: amplifier input voltage
$v_2(t)$: amplifier output voltage
$\theta(t)$: motor shaft position

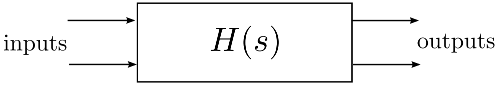
How can we represent the system using a block diagram ?

# Block diagrams

→ Represent the relationship of a system variables graphically.

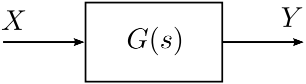
→ Example: The relation between the input voltage and and the position of a DC motor



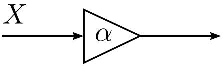



## Basic building elements

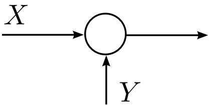
Transfer function




Gain

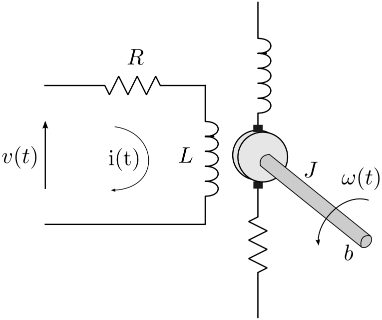



Sum



# Block diagram of a DC motor

→ Electric circuit characteristics

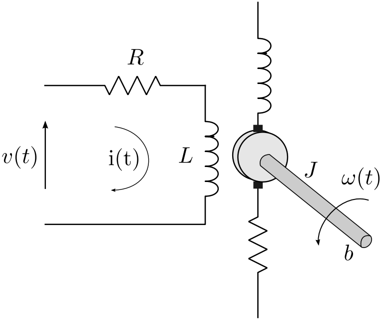



→ Back electromagnetic force voltage

$$V(s) = (R + Ls)I(s) + \omega(s)k_m \rightarrow I(s) = \frac{V(s) - V_m(s)}{R + Ls}$$

# Block diagram of a DC motor

→ Mechanical characteristics




→ Torque constant

$$T(s) = (Js^2 + bs)\theta(s) + T_d \rightarrow \theta(s) = \frac{I(s)k_i - T_d}{Js^2 + bs} \rightarrow \omega(s) = \frac{I(s)k_i - T_d}{Js + b}$$

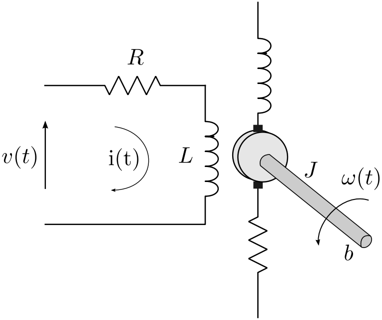
# Block diagram of a DC motor

$$I(s) = \frac{V(s) - V_b(s)}{Ls + R}$$

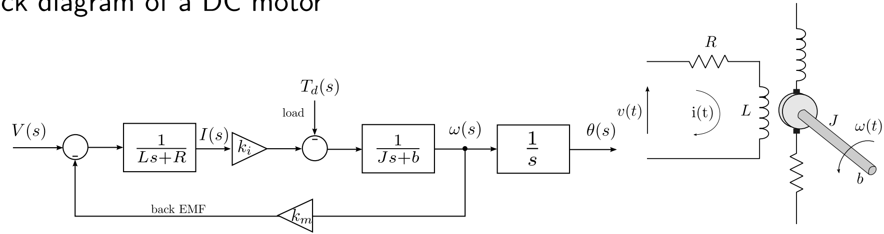
$$\omega(s) = \frac{T(s) - T_d(s)}{Js + b}$$

$$T(s) = k_i I(s)$$

$$V_m(s) = k_m \omega(s)$$



## Block diagram of a DC motor

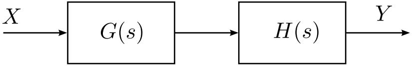



Simulation with Matlab - Simulink

Evaluate the step response of the motor

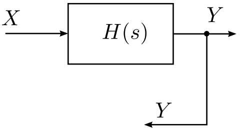
# Basic operations

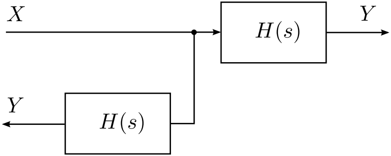
Combining blocks in cascade





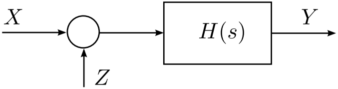
Moving a pickoff point ahead of a block

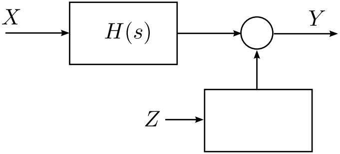


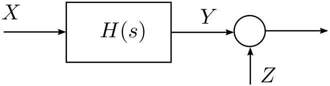
## Basic operations

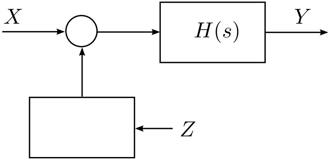
Moving a summing point ahead a block



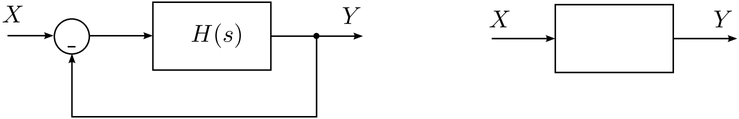




Moving a summing point behind of a block

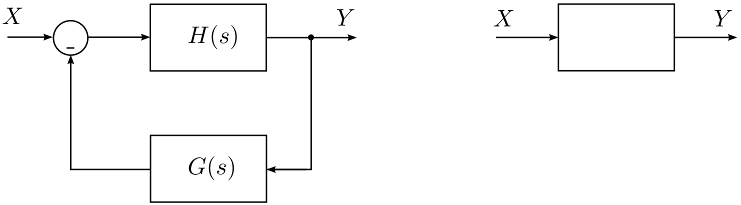


# Eliminating a feedback loop

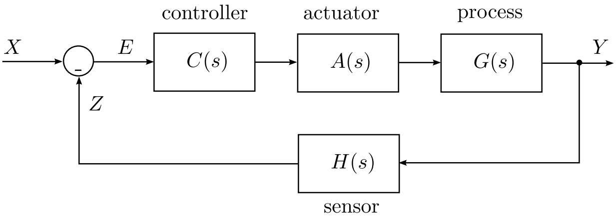


# Eliminating a feedback loop

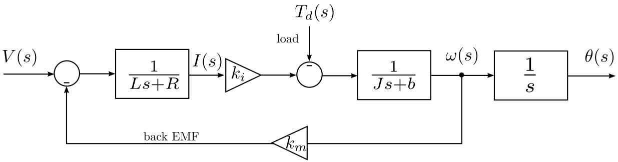


## Example 1

Find the open-loop transfer function of the closed-loop system shown.

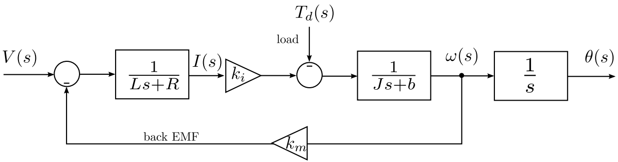


# Example 2 - DC motor




If $T = 0$, what is the transfer function $\theta(s)/V(s)$ ?

## Example 2 - DC motor




$$G(s) = \frac{\theta(s)}{V(s)} = \frac{k_i}{s[(Ls+R)(Js+b)+k_ik_m]} \quad (1)$$

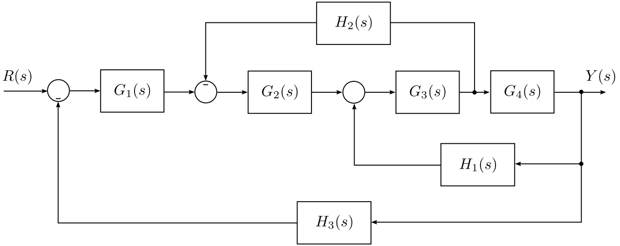
Sometimes the armature time constant $\tau_a = L/R$ is negligible, thus

$$G(s) \approx \frac{\theta(s)}{V(s)} = \frac{k_i}{s[R(Js+b)+k_ik_m]} = \frac{k_i/(Rb+K_iK_m)}{s(\tau s+1)} \quad (2)$$

where $\tau = \frac{RJ}{Rb+K_iK_m}$

# Exercise 23

Find the transfer function $Y(s)/R(s)$ of the system shown.

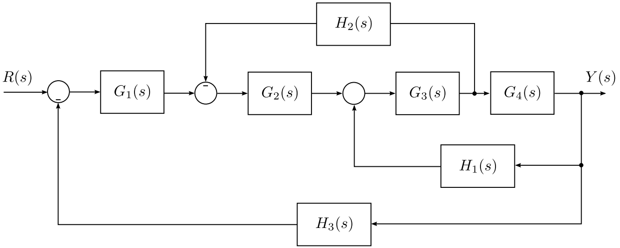


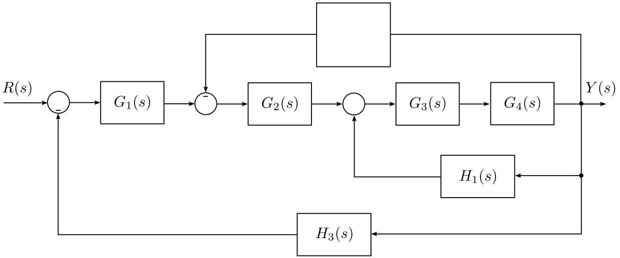

**Procedure**:

$\rightarrow$ Simply the block diagram

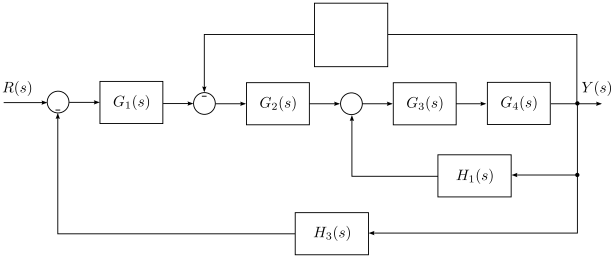
$\rightarrow$ Calculate the closed-loop transfer function

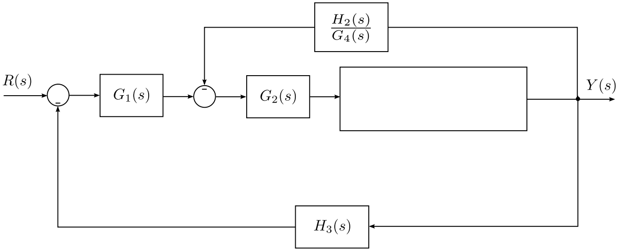
# Exercise 23 - continued



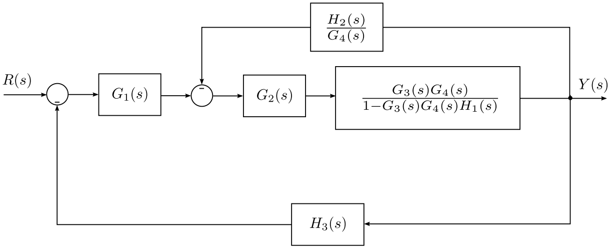



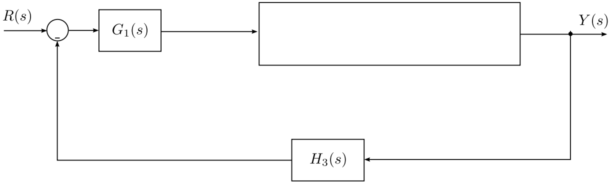
# Exercise 23 - continued



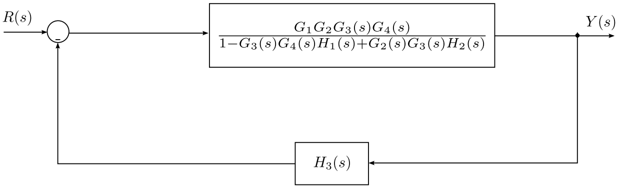



# Exercise 23 - continued



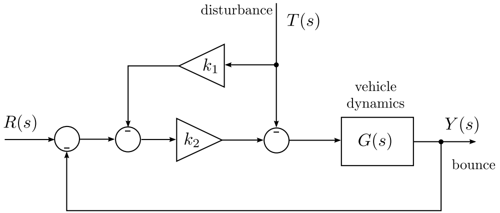



# Exercise 23 - continued



## Exercise 24

An active suspension system can be controlled by a sensor that looks ahead at the road conditions. An example that can accommodate road bumps is shown in the figure. Find the gain $k_1$ so that the vehicle does not bounce when the desired deflection is $R(s) = 0$ and the disturbance is $T(s)$.



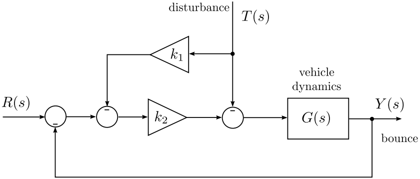

**Procedure**:

→ Find the transfer function from $T(s)$ to $R(s)$

→ Set the bounce to zero ($Y(s) = 0$)

→ Calculate $k_1$

## Exercise 24 - continued



# Exercise 25

The position control system for a spacecraft platform is governed by the following equations:

$$\frac{d^2p(t)}{dt^2} + 2\frac{dp(t)}{dt} + 4p(t) = \theta(t)$$

$$v_1(t) = r(t) - p(t)$$

$$\frac{d\theta(t)}{dt} = 0.5v_2(t)$$

$$v_2(t) = 8v_1(t)$$

$r(t)$: desired position
$p(t)$: current position
$v_1(t)$: amplifier input voltage
$v_2(t)$: amplifier output voltage
$\theta(t)$: motor shaft position

**To do**:

→ Sketch a block diagram of the system

→ Find the transfer function $P(s)/R(s)$

## Exercise 25 - continued

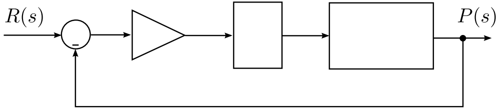
$$\frac{d^2p(t)}{dt^2} + 2\frac{dp(t)}{dt} + 4p(t) = \theta(t)$$

$$v_1(t) = r(t) - p(t)$$

$$\frac{d\theta(t)}{dt} = 0.5v_2(t)$$

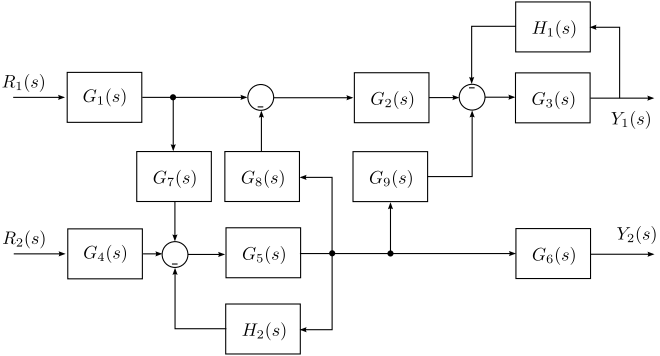
$$v_2(t) = 8v_1(t)$$

## Exercise 25 - continued

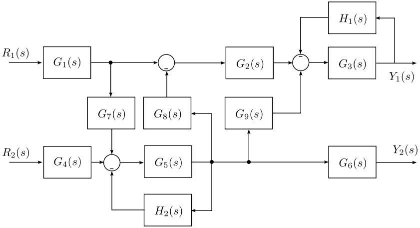


# Exercise 26

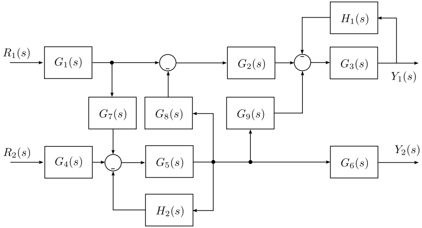
Compute the transfer function $Y_1(s)/R_2(s)$. Hint: Using the principle of superposition, set $R_1(s) = 0$.



# Exercise 26 - continued

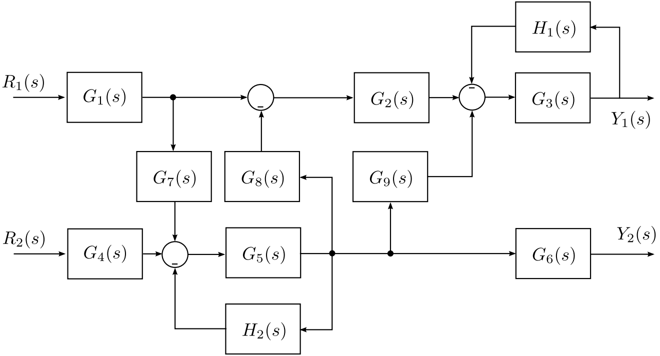


# Exercise 26 - continued

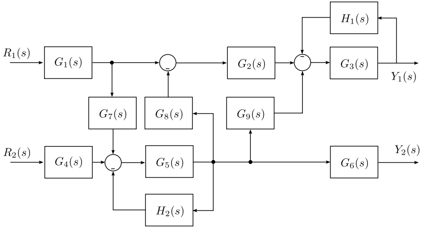


# Exercise 27

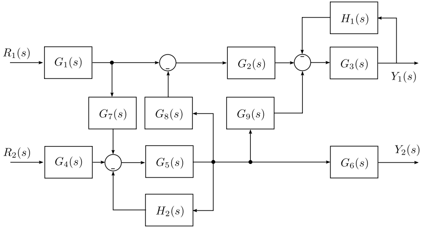
Compute the transfer function $Y_2(s)/R_1(s)$. Hint: Using the principle of superposition, set $R_2(s) = 0$.



# Exercise 27 - continued

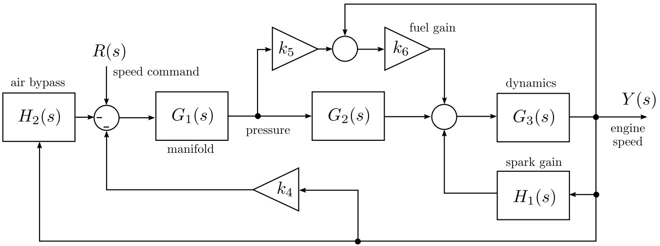


# Exercise 27 - continued

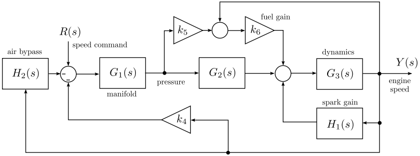


# Exercise 28

Compute the transfer function $Y(s)/R(s)$ for the idle-speed control system for a fuel-injected engine as shown in the block diagram.



# Exercise 28 - continued

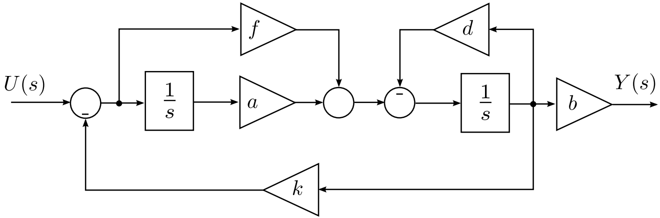


# Exercise 28 - continued

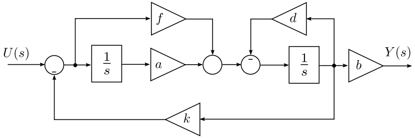
# Exercise 28 - continued

## Exercise 29

Compute the transfer function $Y(s)/U(s)$ for the block diagram shown.

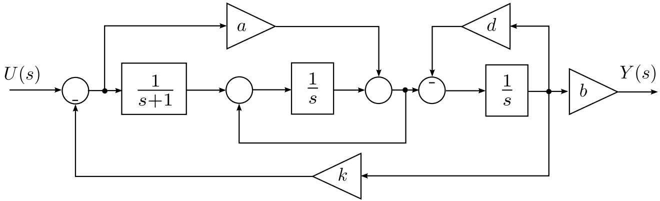


# Exercise 29 - continued



# Skills check 17 - From 2018 midterm examination

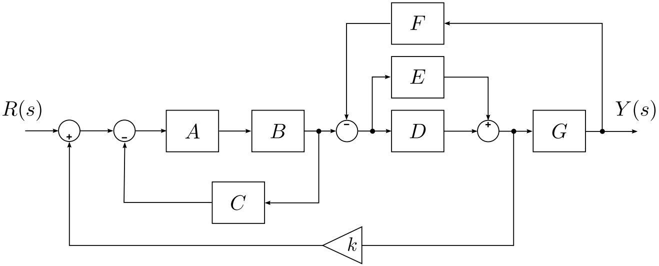
Find the transfer function $Y(s)/U(s)$[1].




[1] Answer is a few slides further.

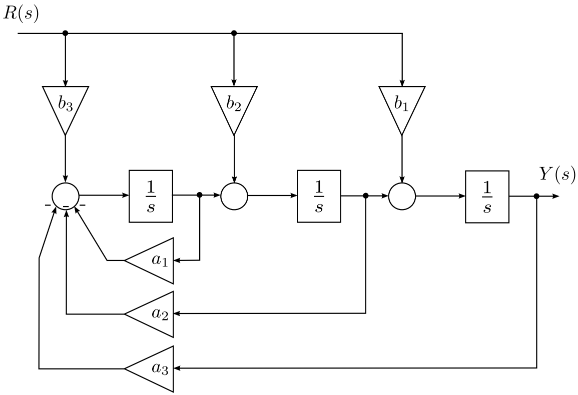
## Skills check 18 - From a past midterm examination

The Shuttle Remote Manipulator System (SRMS) or Canadarm was a joint venture between the governments of the United States and Canada to supply the NASA Space Shuttle program with a robotic arm for the deployment or retrieval of space hardware from the payload bay of the orbiter. If $R(s)$ and $Y(s)$ are the commanded and actual positions of the arm's end-effector, find the transfer function $Y(s)/R(s)$.



# Skills check 19 - From a final examination

Find the transfer function $Y(s)/R(s)$.



## Answers to skills check

Skills check 17

$$\frac{Y(s)}{U(s)} = \frac{a(s+1)s+1}{(s^2-1)(s+d)+k[a(s+1)s+1]}b$$

Skills check 18

$$\frac{Y(s)}{R(s)} = \frac{ABG(D+E)}{(1+ABC)([1+GF(D+E)]-ABK(D+E)}$$

Skills check 19

$$\frac{Y(s)}{R(s)} = \frac{b_3+b_2(s+a_1)+b_1(s^2+a_1s+a_2)}{s^3+a_1s^2+a_2s+a_3}$$

# Next class...

- Steady state error